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THE UNIVERSITY OF ALBERTA

A STUDY OF THE APPLICATION OF LINEAR PROGRAMMING  
AND THE ASSOCIATED TECHNIQUE OF DECOMPOSITION  
FOR USE ON FARM MANAGEMENT PROBLEMS IN ALBERTA

by

ALLAN WAYNE ANDERSON

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES  
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE  
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DEPARTMENT OF AGRICULTURAL ENGINEERING

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UNIVERSITY OF ALBERTA  
FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Study of the Application of Linear Programming and the Associated Technique of Decomposition for Use on Farm Management Problems in Alberta" submitted by Allan Wayne Anderson in partial fulfilment of the requirements for the degree of Master of Science.



## ABSTRACT

The principal objective of this project was to examine the problems encountered in preparing linear programs for Alberta farmers. Thus data were collected from twelve farms distributed throughout the Province of Alberta and assembled into linear programming matrices, using established procedures.

The major problem encountered was in the substantial time required to prepare and process the linear programs. A learning curve was established for the programmer regarding program preparation time which showed that the first program required about 2.25 weeks actual working time to prepare, while the last few programs required about 1.0 week actual working time to prepare. No such curve was observed for the program processing time. This time varied about an average value of 3.0 weeks elapsed time regardless of experience. Actual programmer working time was approximately 1.0 week. Most of this processing time consisted of waiting in line to be processed on the computer which was necessitated by the large number of trials on each program required by the programmer. Clerical time for preparing the data for the computer was not found to be very large nor was the computer time to solve the problem. Another substantial problem encountered was the tedious nature of preparing these programs and it is suggested that this is probably the obstacle which has kept linear programming in the area of research and benchmark studies.

The matrices (prepared as they were for this project) exhibited the characteristics which allowed the problem to be solved using the decomposition technique. It is suggested that if the problem can be solved in parts, it should be possible to put the matrix together in



sections, and if so this offers substantial opportunity to use "standard" sections for different areas of the province. Standardized procedures could also reduce substantially the amount of tedious work involved. A suggestion was also made that the two problems mentioned above would not be as critical over an extended period of time if the program was kept and processed every year for the farmer. The adjustments required to update a program from year to year require very little time and thus the original matrix preparation time could be done infrequently on a per farm basis.

A third problem encountered was that of the large distances encountered in Alberta. This means direct communication with the individual is difficult and the transfer of information often suffers. As well, misunderstandings do occur.

The programs prepared for this project followed the British example. While this posed a problem in mastery of the technique, matrices prepared following British examples were felt to offer greater flexibility than the majority of American examples, especially since computer capacity was not limiting.

The supply of data which could be used to obtain matrix coefficients was plentiful and the Alberta farm can be well represented by the mathematical model required for linear programming. Although only a limited analysis was carried out on the farmers' response, the farmers in the study were, in general, satisfied that the results were valuable to them. Thus it was concluded that linear programming should eventually be available to the Alberta farmer as a service. The implementation of a linear programming service may be facilitated by the use of standardized procedures and prepared sub-programs.



## ACKNOWLEDGEMENT

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The services of the Farm Economics Branch, Alberta Department of Agriculture are gratefully acknowledged. Association with the personnel of this agency was pleasant and rewarding. Also, the co-operation and interest shown by the twelve farmers in the study and the District Agriculturists is sincerely appreciated.

Sincere thanks are in order to Mrs. R. Kerr who did such a capable job of arranging and typing the manuscript. This was a formidable task with the large amount of data in tabular form. As well, sincere thanks to my wife, Norma, for the assistance in proof-reading and re-writing the first draft.

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## 1. INTRODUCTION AND OBJECTIVES

Linear programming has been one mathematical technique which has received a great deal of interest in the past decade. Most of the work has been done in the United States, especially in Iowa under E.O. Heady<sup>26</sup>. A great volume of publications from this area was published in the mid 1950's when the initial interest was high. However now the numbers of publications concerning the topic have been reduced to relatively few per year, and many of the publications only use the technique for illustrating other points. There has also been a substantial amount of work from Britain in the last eight to ten years, with McFarquhar<sup>32</sup>, Stewart<sup>45</sup>, and Edwards<sup>17</sup> being examples. In Canada, on the other hand, relatively few publications can be found that have any reference to linear programming on the Canadian scene. There is some work done by Bell<sup>5</sup> in Saskatchewan, by Bowland<sup>9</sup> in Alberta, and by Gilson, Yeh, and Hodgson<sup>21</sup> in Manitoba for balancing livestock rations. Linear programming in the context of classical theory was examined by Gilson<sup>20</sup> in Manitoba. As well, the technique is taught as part of Canadian university courses and used to a certain degree. However, work with the technique is not extensive in Canada at the present time. In Alberta, up to the time of the preparation of this thesis, little work had been done to use linear programming to help farmers in making decisions regarding the optimum combinations of enterprises on their farms.

### 1.1 DISTRICT OF WORK

Alberta has a wide variation in farming conditions and farm firms. To facilitate explanation of the dominant farming patterns in the province, it has been divided into regions shown in Figure 1. The map is based on



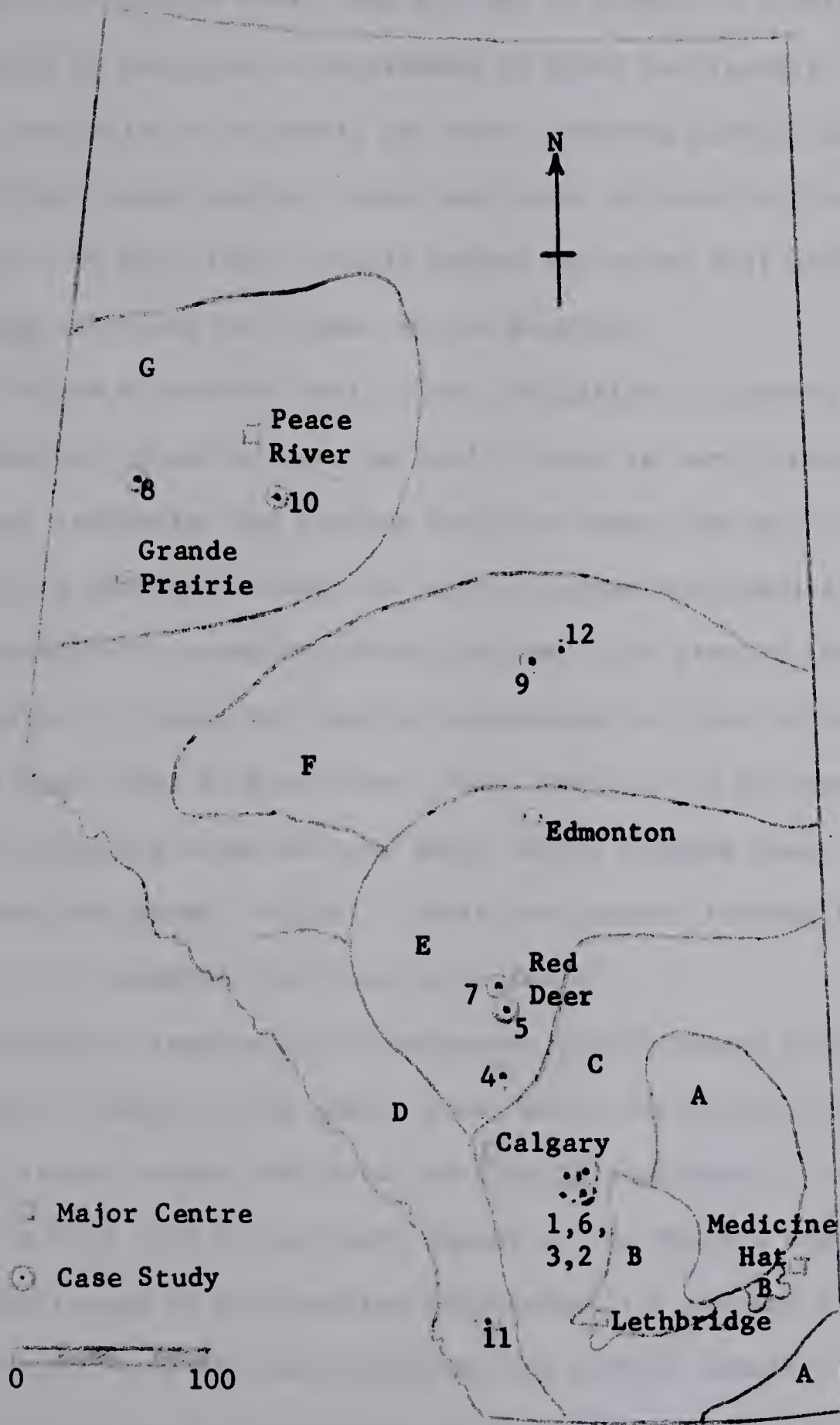


Figure 1. Map of Alberta Showing the Division of Alberta into the Predominant Farming Patterns and the Location of the Case Studies.



the maps found in the Alberta Farm Guide<sup>2</sup> on Alberta soil zones and Alberta irrigation areas, and the map of Alberta<sup>50</sup> showing the townships surveyed by the Alberta Department of Lands and Forests.

The region A is mainly the drier ranching portion of the province. Individual farms are very large and there is very little crop production. Rainfall is very light in this region and brown soil predominates. Grazing rates are the lowest in the province.

Region B includes most of the irrigation in Alberta as well as a substantial amount of grazing land. There is very little crop production without irrigation and grazing rates on range land are low. Several community pastures within the region provide irrigated farms with the opportunity for greater diversification. The size of farms under irrigation is small but capital investment per farm varies from moderate, where cash crops such as wheat, flax, barley, and hay are grown, to high where specialty crops such as sugar beets, canning peas, carrots, and potatoes are grown. As well, there are several ranches in this region which are comparable to those in region A.

Region C represents the extensive grain farming portion of the province. Much of this region grows wheat exclusively with holdings being large in both land area and capital investment. Production per acre on this land is not high, except on the Western edge, but the acreage farmed by one operator compensates for the low yield.

Region D is the foothills ranching area of Alberta. This land is usually unsuited to extensive crop production due to altitude, topography, and soil type. Grazing rates however, are relatively high due to adequate precipitation. The area of land under control of each operator is large



and capital investment per ranch is relatively high. This region includes many of the long established ranches.

Region E is one mixed farming portion of the province and includes some of the most productive land in the province. A large percentage of the land is cultivated, with grains and forage crops being the predominant crops. Due to the substantial amount of forage and coarse grains raised, livestock numbers are high. The acreage per farm is somewhat less than in region C, but capital investment per farm is comparable.

North central Alberta, shown as region F on the map, is another mixed farming region of the province. Farms tend to be smaller in size and capitalization than in region E, especially along the northern edge of the region. Due to more adequate rainfall, production per acre is relatively high, providing the recommended cultural practices are followed.

The Peace River district, shown as region G, is a relatively new region of the province. Emphasis is on forage seed and swine production, with grain and beef also raised. Production per acre is usually high, but farm capitalization tends to be low in much of the region.

This classification largely ignores the city milk sheds. These are found around each of the cities and the area depends upon the population served. The case studies did not include a dairy farm or an irrigated farm. The classification has been made to show the wide variety of conditions found in Alberta to which linear programming may be applied. It is felt that, whereas no statistical procedures were used to insure the selection of representative farms in each area, a representative series of farms were programmed. The reasoning was based on the case studies being throughout Alberta, the farmers being selected from local groups, and discussion with the District Agriculturist of the area.



## 1.2 OBJECTIVES

The objectives of this project are tabulated as follows:

- a) To examine the time spent per farm from the initiation of a program\*, to the presentation of results.
- b) To examine the problem of communicating over the large distances found in Alberta while preparing linear programs.
- c) To examine the availability and the sources of data for Alberta farming.
- d) To examine the work load associated with subsequent program updating.
- e) To examine the technique of decomposition to spread the work load in the preparation of large numbers of characteristic farm programs.
- f) To examine the feasibility of using a standardized form to help in the preparation of large numbers of linear programs.

## 1.3 DEVELOPMENT OF LINEAR PROGRAMMING

Extensive interest in linear programming has developed since Dantzig<sup>15</sup> formulated a practical method of solution in 1947. The algorithm for the solution, plus the post World War II development of high speed electronic digital computers have resulted in linear programming techniques being applied to many and varied problems. Dantzig<sup>14</sup>, in his text, gives a relatively complete resume of linear programming to 1960. He credits the early mathematician, J.B.J. Fourier, with being the first individual to appreciate the basic mathematical concept.

Since 1947, a tremendous amount of literature on linear programming and its extensions has been published. Candler and Musgrave<sup>11</sup> prepared a partial bibliography to 1960 listing 52 references, mainly from official publications and primarily limited to agricultural areas. There are also a number of graduate theses on the topic which are listed in

\* Unless otherwise stated, program refers to a completed linear programming matrix while programmer refers to one who prepares a program.



abstracts, but are not readily available to individuals for reference. As well, there are probably many unpublished mimeographed articles. A chronology of the development was given by Eisgruber and Reisch<sup>19</sup> in a tabulation of the extent of teaching of the technique. They note also that prior to 1958, most post graduate work on the technique was done by Ph. D. candidates while after this date, most post graduate work on linear programming is done by M. Sc. candidates.

Many extensions to linear programming have been formulated including integer programming by Gomory<sup>23</sup>, dynamic programming by Bellman<sup>6</sup>, quadratic programming by Markowitz<sup>36</sup> and Wolfe<sup>51</sup>, decomposition by Dantzig and Wolfe<sup>16</sup>, and others. In addition, many diverse problems have been made amenable to linear programming by a number of authors. Examples are MacFarquhar<sup>32</sup> with risk and uncertainty, and MacHardy<sup>34</sup> with inclusion of the capital plant in determining the optimal farm plan.

As can be concluded from the above, the linear programming field is a large one, and it is not the object of this thesis to cover the whole field. This study will be concerned principally with:

- a) the resources required to prepare linear programs for optimizing enterprise selection on selected Alberta farms and
- b) some considerations which might reduce the amount of work in the preparation of linear programs for individual farm firms.



## 2. OUTLINE OF THE PROJECT

### 2.1 ORIGIN

In 1962, the Farm Economics Branch of the Alberta Department of Agriculture was considering offering a service of linear programming to Alberta farmers. At the same time, Dr. F.V. MacHardy, Head of the Department of Agricultural Engineering, University of Alberta, had developed considerable interest in the technique. Discussion between the Departments ensued and it was decided to conduct a co-operative pilot project. The Department of Agricultural Engineering agreed to supply the personnel to do the actual linear programming and arrange for use of the computing facilities of the University, while the Farm Economics Branch agreed to provide channels to reach prospective farmers. This thesis constitutes a record of this project.

### 2.2 DATA COLLECTING AND PROCESSING PROCEDURE

The Farm Economics Branch had established, within the province, a network for the analysis of farm records. The first step of this network consisted of local farm management groups organized by the District Agriculturist for the area. The groups consisted of district farmers interested in having their farm records analyzed and willing to complete the necessary forms supplied by the Farm Economics Branch. The completed forms were sent by the District Agriculturist to the Farm Economics Branch for analysis. When the analysis was completed, the results were returned to the farmer through the District Agriculturist. This network was considered to offer the best means to contact prospective farmers and to communicate with them during this linear programming study.



The first stage of the project consisted of obtaining the names of farmers interested in having their farms linear programmed. The Farm Economics Branch performed this task by asking the District Agriculturists' co-operating with them on the farm record analysis to submit the names of one or two farmers from their farm management groups who would be interested in having their farms linear programmed. There were primarily four advantages of using this method in contacting the farms, which are as follows:

- a) The farmers contacted would already have relatively complete records of recent farm performance.
- b) These farmers would not be unfamiliar with making estimates which could be verified in performance.
- c) These farmers would be accustomed to answering relatively personal questions posed by Extension personnel and would give accurate answers.
- d) The District Agriculturist could supply much information about the area (i.e. average yields, feasible agronomic practices) which the individual farmer may not be able to supply.

The second stage of the project consisted of a visit to the farmer by the author and one member from the Farm Economics Branch to explain the project and obtain the necessary data from his farm. Data were obtained to enable twelve Alberta farms to be linear programmed. These farms were located in the following regions:

Region C - four farms

Region D - one ranch

Region E - three farms

Region F - two farms

Region G - two farms



The locations of these farms are illustrated in Figure 1. As can be noted, this gives reasonably good distribution through the province.

The third stage consisted of the actual program preparation and solving. Problem formulation and clerical work was done in the Department of Agricultural Engineering. Computer facilities to solve the problems were available from the Department of Computing Science, University of Alberta.

The fourth and final stage consisted of returning the solution to the farmer. This was accomplished by the author sending a written report of the solution and the data used in arriving at the solution to the farmer via the District Agriculturist. The report included a request that the farmer reply with any modifications he would like to have made in the data used or the activities if he felt the program was not representative of his farm. If no changes were requested, this completed the case study. Otherwise any program modifications submitted by the farmer or the District Agriculturist were made, the problem was resolved, and the new results were returned. This was repeated as many times as necessary. As well, a copy of all reports sent out to the farmer was sent to the Farm Economics Branch.

### 2.3 PROGRAM FORMULATION - THE FUNCTIONAL

The first step in the preparation of a linear program is to determine the function to be maximized. In the majority of the British publications (e.g. MacHardy<sup>34</sup> and Stewart<sup>45</sup>) the functional maximizes what is called the farm's "gross margin". The term comes from the "gross margin analysis" technique which was developed in Great Britain as another farm planning tool. MacHardy<sup>34</sup> gives credit, via references, to Peart and Rowbottom, to



V. Liversage of Northern Ireland, and to the B.B.C. and Farm Economics Branch, Cambridge University, for its initial development. MacHardy<sup>34</sup> summarizes the logic of Peart and Rowbottom and justifies the use of gross margin for linear programming on the basis of the following:

- a) The gross margin technique for farm planning has many of the concepts underlying the data presentation for linear programming.
- b) Peart and Rowbottom list one of the uses of the gross margin technique as being a forward planning tool to guide in the choice of enterprises, and the scale of each.

MacHardy<sup>34</sup>, on this basis, defines gross margin as the difference between the gross revenue and variable costs and gives the analysis in mathematical terms. Most American literature, (Love<sup>30</sup>, King, Bishop and Sutherland<sup>29</sup>, Peterson<sup>42</sup>) maximize the returns to labor, capital and management. In essence, this is the gross returns from an enterprise less the operating costs. Essentially then, the same function is maximized in both American and British examples but the name is different. Since the American literature did not establish a name for the function being maximized, it was decided to follow the British example and maximize the farms' "gross margin" in order to establish a term which, once explained to the farmer, could be discussed with little confusion.

#### 2.4 PROGRAM FORMULATION - THE COEFFICIENT MATRIX

The initial step in the preparation of the matrix was to get a general outline of the farm in the form of linear mathematics. This consisted of arranging the necessary constraints and preparing a list of activities, and was usually a combined procedure done on specially prepared matrix sheets. The restrictions were placed in the rows and the activities



listed on the column headings. These matrix sheets were the same format as those illustrating the programs in section 3. Matrix coefficients that did not require calculation were entered in the matrix at this time as well. The primary purpose however, was to determine the linear equations necessary to represent the farm enterprise.

The matrices prepared in this manner were similar to the matrices in British examples such as those prepared by Stewart<sup>45</sup> and MacHardy<sup>34</sup>, in that the crop rotation or the beef ration was not definitely selected prior to solving the program. Much of the crop rotation and the beef ration was selected by solving the problem. This is not the same as the matrices in American examples such as those prepared by Love<sup>30</sup>, Bishop<sup>8</sup>, and Heady and Candler<sup>27</sup>, where activities require complete selection of feeding programs and crop rotations.

The difference between the two procedures is best illustrated by an example. Suppose it was desired to prepare a program for the cropping system on the farm. The method most often seen in the British examples appears as in Figure 2. The cropping restrictions are based on the necessary agronomic practices of the area.

Most American authors have used the procedure illustrated in Figure 3. As can be observed from Figure 3, the number of different rotations depends on how many the programmer wishes to place in the program. There are advantages to both procedures. The procedure illustrated in Figure 2 offers a great deal more flexibility, as the optimum rotation is selected within the program. Also the programmer does not need to do as much calculation. More coefficients are required in the matrix, but these are either coefficients which may be entered without calculation, or



			Crop A (Acres)	Crop B (Acres)	Crop C (Acres)
Acres of Land	$b_1$	$\geq$	1	1	1
Maximum Acres of Crop A	$b_2$	$\geq$	1		
Minimum Acres of Crop B	$b_3$	$\leq$		1	
*Balance of Crop A to Crop C	0	$=$	$a_{41}$		$a_{43}$
Functional (Maximize)	Z	$=$	$C_1$	$C_2$	$C_3$

\*One number of  $a_{41}$  or  $a_{43}$  is negative.

Figure 2. Illustration of a Cropping Matrix Most Often Seen in British Examples.

			Crop A (Maximum)	Crop B (Minimum)	Crop B Only	Crop A ( $\frac{1}{2}$ Maximum)	Crop C (In Proportion to Crop A)	Crop A ( $\frac{1}{2}$ Maximum)	Crop C (In Proportion to Crop A)
			Crop C (In Proportion to Crop A)			Crop B (Remainder)		Crop B (Remainder)	
Acres of Land	$b_1$	$\geq$	1	1	1	1	1	1	1
Functional (Maximize)	Z	$=$	$C_1$	$C_2$	$C_3$	$C_4$	$C_3$	$C_4$	$C_4$

Figure 3. Illustration of Method Used by Most American Authors to Select the Crop Rotation.



they are coefficients which are used in calculating the cost coefficient ( $c_i$ ) for the method shown in Figure 3. The advantage of the method in Figure 3 is that, by selecting the most likely rotations, the programmer can reduce the size of the program. In early linear programs this was a distinct advantage, as either computers were not available or the computers available had limited capacity. However, with the advent of high capacity, high speed electronic computers this has become less important.

Once the outline had been prepared, it was necessary to fill in those coefficients which required substantial calculating. Cost row coefficients, as one might expect, often were not true gross margins as many activities with saleable end products were represented by a combination of activities listed in the program.

## 2.5 OBTAINING THE SOLUTION

Once the matrix had been completed from the data collected from the various sources, it was scaled to improve the computer accuracy. The scaled matrix was then punched on cards in a format specified by the particular computer program that was used and given to the Department of Computing Science, University of Alberta, for processing. Since problems were processed by the Department of Computing Science in the order in which they were received from the various departments in the University of Alberta, a slight delay occurred before the solution was obtained from this department. The computer solution was then decoded from the numerical computer form to the statement of the optimum enterprise combination. The equipment used to solve these problems varied. Seven problems were solved using an IBM 1620 computer and computer programs prepared by C.R. Nichols<sup>39</sup> (modifications by K. Kreiger<sup>39A</sup> and by



A. Nickels and L. Davis<sup>39B)</sup> and by F.W. Wood.<sup>52, 53</sup> Five problems were solved using an IBM 7040 computer and a computer program prepared by Clasen<sup>13</sup>.

## 2.6 THE USE OF DECOMPOSITION

This technique was developed in 1960 by Dantzig and Wolfe<sup>16</sup>, and the information for this thesis was taken from Dantzig's text<sup>14</sup>. A detailed explanation of the technique is given in the appendix. The work in this thesis demonstrates how farm linear programs, which have been prepared in the manner used for this project, fit the technique of decomposition. Investigation of the technique concentrated on its use for reducing the labor in program preparation rather than for actually solving the problems.



### 3. CASE STUDIES

In general, the case studies are discussed in the order in which they were completed. The only exception is Case 12 where the matrix was prepared by Dr. F.V. MacHardy<sup>35</sup>. The basic data and the original matrix are included in the exposition on each case study, but only Case 1 shows the detailed calculations required in obtaining a complete set of matrix coefficients. However, the same basic procedure was used for all case studies. The only differences between case studies lie in the original data and in modifications to improve the reliability of matrix coefficients. Therefore recording the calculations used for each case study would involve unnecessary repetition.

All the farmers contacted were skilled farm managers and it was felt that they could easily handle any program which was the solution to the problem, provided all activities were suitable to the area.

#### 3.1 CASE 1

##### 3.1.1 Resources.

The farmer in Case 1 had the following resources available:

- a) Land
  - Arable - 1,585 acres
  - Pasture - 6,000 acres
- b) Livestock
  - Cow herd - 130 head
  - Feedlot capacity - 150 head of yearling steers at one time
- c) Capital
  - Land and Building Purchase - \$85,000.00
  - Livestock Purchase - \$36,000.00
- d) Labor Force - Operator plus one full time hired man



### 3.1.2 Farmer's Preferences.

The operator did not wish to seed more than 100 acres of flax and wanted at least one third of the land used for grain or cash crop production left as summerfallow. He was interested only in enterprises involving beef cattle, grain production, and cash crop production.

### 3.1.3 Data Inputs.

#### 3.1.3.1 Crop Yields and Prices.

The yield estimates were a combination of the farmer's estimate and the District Agriculturist's estimate. Prices were obtained from the farmer or his farm records with the Farm Economics Branch. The cropping data are summarized in Table 1.

Table 1. Crop Yields and Prices Used in Case 1.

Crop	On Land Summerfallowed the Previous Year	On Land Cropped the Previous Year	Farm Price
Hard Red Spring Wheat	21 bus/acre .5 ton/acre straw	14.5 bus/acre .35 ton/acre straw	\$ 1.50/bu
Durum Wheat	21 bus/acre		\$ 1.75/bu
Flax	15 bus/acre		\$ 3.00/bu
Oats	44 bus/acre .8 ton/acre straw	30 bus/acre .6 ton/acre straw	\$ .55/bu
Barley	30 bus/acre	20 bus/acre	\$ .80/bu
Hay (reseed 1 acre in 5)	Yield - .5 ton/acre/yr.		\$20.00/ton



### 3.1.3.2 Field Machinery.

In order to get the labor and cost figures for cropping activities, a list of the farmer's equipment was required. This included the model or size of the machine and occasionally, the original cost. Usually the original cost was obtained from an equipment dealer. Table 2 gives the equipment data pertinent to Case 1.

Table 2. Equipment Used by the Farmer in Case 1.

Machine	Size or Model	Original Cost
Tractors	660 I.H.C. Diesel	\$5,800.00
	450 I.H.C. Gasoline	\$4,300.00
	300 I.H.C. Gasoline	\$3,200.00
Noble Blade	16 ft	\$1,300.00
Cultivator (rodweeder attachment)	16 ft	\$ 900.00
Rodweeder	20 ft	\$1,200.00
Single Disc	20 ft	\$ 400.00
Oscillating Harrows	21 ft	\$ 600.00
Drill	14 ft	\$1,200.00
Discer	15 ft	\$2,000.00
Mower	7 ft	\$ 600.00
Rake	12 ft	\$ 650.00
Farmhand Front End Loader for Tractor		\$1,600.00
Swathers	16 ft	\$1,600.00
	15 ft	\$1,550.00
Combines	I.H.C. 141 (self-propelled)	\$6,300.00
	I.H.C. 140 (P.T.O.)	\$3,800.00



### 3.1.3.3 Livestock.

The basic price and weight data for the beef enterprises came from an examination of the farmer's records with the Farm Economics Branch in Edmonton, records of livestock prices on the Calgary stockyards which were obtained from H.C. Love<sup>31</sup>, and the farmer's estimate for certain classes. Table 3 gives the summary of this data.

Table 3. Livestock Prices and Weights Used for Case 1.

Class	Buying			Selling		
	Price(¢/lb)	Wt(lb)	Gross(\$)	Price(¢/lb)	Wt(lb)	Gross(\$)
Calves off cow in fall	-	-	-	26.0	450	\$117.00
Calves fattened over winter	26.00	450	\$117.00	23.25	850	\$197.63
Calves wintered	26.00	450	\$117.00	24.0	600	\$144.00
Yearlings on pasture	24.00	600	\$144.00	22.0	800	\$176.00
Yearlings fattened in summer	24.00	600	\$144.00	24.0	950	\$228.00
Long yearlings fattened in winter	22.00	800	\$176.00	22.7	1,100	\$249.70

### 3.1.3.4 Farming Practice.

On this farm, land in fallow the previous year was seeded in late April and May with the discer, followed by the harrows. In wet springs, this was preceded by a tandem disc and harrows operation. Fertilizer was applied with the seed at the rate of 40 lb/acre of 11-48-0 to flax and oats on fallow, 45 lb/acre of 16-48-0 to durum wheat and 65 lb/acre of 16-48-0 to oats on stubble. To seed a crop on land previously cropped, an extra



operation of cultivating the land in April and prior to seeding in May was required. The only stubble crop (i.e. seeded on land which had been seeded to a cash crop of grain or flax the previous season) fertilized was oats as the operator felt that the other crops did not respond. All crops were sprayed using custom work in June, and swathed in late August. Combining was done in late August and early September. Tame hay was harvested in June and July with the sequence being mow, rake, and stack with the front end loader. The rate of stacking was 25 ton per day and required two men. Summerfallow received three cultivations; cultivate in May, cultivate with rod weeder attachment in July, and rodweed in September. The stocking rate of native range was 40 acres per cow where 25 acres were used for summer grazing and 15 acres for winter grazing. He also had the opportunity to rent winter pasture relatively close to the home farm for \$3.00/cow/month. In the program, it was assumed that the cows and any growing animals were put on pasture during the summer. Thus there was no summer ration requirement as it was replaced by a pasture acreage requirement. Otherwise the ration was selected from the various available feeds.

#### 3.1.3.5 Miscellaneous Data.

In addition to the above information, a certain amount of information on minor cost items was required. The data used in Case 1 are as follows:

- a) Fuel costs - Gasoline - \$ .223/Imperial gallon  
                  - Diesel - \$ .181/Imperial gallon
- b) Labor costs - Regular labor - \$ .90  
                  - Casual labor - \$1.80



- c) Fertilizer cost - \$110.00/ton - 16-48-0
  - \$105.00/ton - 11-48-0
  - \$ 70.00/ton - 16-20-0
- d) Straw - \$8.00/ton
- e) Seed costs -
  - Durum wheat - \$3.00/acre
  - Red Spring wheat - \$2.40/acre
  - Oats - \$1.50/acre
  - Barley - \$1.75/acre
  - Flax - \$4.00/acre
  - Winter wheat - \$3.00/acre
  - Grass - \$3.60/acre reseeded
- f) Custom spraying - \$ .70/acre
- g) Miscellaneous livestock data -
  - Trucking costs - \$60.00/load of 25 1,100 lb steers
  - Labor - 6.4 months for 133 cows with calves
    - 3.2 months for 98 head of yearlings fattened
  - 32% beef cattle supplement - \$90.00/ton
  - Beef supplement - \$60.00/ton (1/3 dehydrated alfalfa, 1/3 dried beet pulp, 1/3 screenings)
- h) Capital cost of additional land -
  - Arable - \$80.00/acre
  - Pasture - \$40.00/acre
- i) Miscellaneous land use -
  - Stubble crop used for winter pasture supplies the equivalent of .2 acres native winter range
  - Cover crop (oats seeded on summerfallow in July)
    - one acre supplies feed for .67 yearling steers for 30 days but the farmer loses the equivalent of 5 bus/acre of wheat in the reduced yield from the crop seeded the following year.

The above are the basic data which were obtained for Case 1. Next the rough program was prepared for a list of the activities and restrictions. Matrix coefficients were entered only if they were obvious. For example, each crop is obviously based on a one acre unit so the entry in the acreage restriction was made. However, some calculation was required to determine the amount of April labor required so it was marked for a



calculation. To illustrate the procedure, the explanation of this case study includes these calculations.

#### 3.1.4 Detailed Calculations.

The initial step was to calculate the machinery operating costs per acre before analyzing the activities on an individual basis. The reason was that cropping activities all required the use of the same machines and calculating these costs separately saved time.

##### 3.1.4.1 Machinery Operating Costs.

###### 3.1.4.1.1 Tractor costs:

The fuel consumption data was based on the information in Implement and Tractor Red Book Issue<sup>28</sup>, where the Nebraska tests are summarized. For tractors tested after 1958, the fuel consumption for drawbar tests was given in horsepower hours per U.S. gallons for:

- a) maximum available power
- b) 75% of pull at maximum power
- c) 50% of pull at maximum power.

In tractors tested prior to 1959, the fuel consumption for drawbar tests was measured in pounds of fuel per horsepower hour at:

- a) maximum available power and at
- b) rated power (75% of maximum drawbar horsepower corrected to standard conditions.)

Thus the procedure for calculating the fuel consumption in Imperial gallons per hour was as follows:

- a) Calculating 75% of the maximum available drawbar power



b) Obtain fuel consumption from the tables

- (1) in horsepower hours per U.S. gallons at 75% of pull at maximum power for tests after 1958.
- (2) in pounds per horsepower hour at rated power for tests prior to 1959.

c) Calculate the Imperial gallons per hour as follows:

(1) For tests after 1958,

$$\frac{75\% \text{ of maximum horsepower} \times .833 \text{ Imperial gallon/U.S. gallon}}{\text{HP hours/U.S. gallon}} = \text{Imperial gallons per hour}$$

(2) For tests prior to 1959,

$$\frac{\text{Pounds/HP hour} \times 75\% \text{ of maximum horsepower}}{7.41 \text{ pounds/Imperial gallon}} = \text{Imperial gallons per hour}$$

The cost for oil and filters was calculated as 15% of fuel cost for gasoline tractors and 20% of fuel cost for diesel tractors. This was based on personal discussion with B.T. Stephanson<sup>44</sup> and on the A.E. yearbook<sup>1</sup>. Unfortunately this factor was neglected in six of the case studies.

#### 3.1.4.1.1.1 Operating cost of I.H.C. 660 diesel:

- Fuel cost

- Data - Maximum drawbar horsepower = 71.38 HP  
(Nebraska tests)

- Fuel consumption @ 75% of pull at maximum power = 12.72 HP hr/U.S. gallon

- Price = 18.1¢/Imperial gallon delivered



- Calculation:

$$\frac{71.38 \times .75 \times .833 \times .181}{12.72} = \$ .635$$

- Oil, grease and filters. (20% of fuel cost) \$ .127
- Repairs - .0001% of original cost/hour used \$ .580  
Original cost - \$5,800.00
- Total operating cost per hour \$ 1.342

3.1.4.1.1.2 Operating cost of I.H.C. 450 (Farmall) gasoline:

- Fuel cost

- Data - Maximum drawbar horsepower = 51.3 HP

- Fuel consumption at rated horsepower = .578 lb/HP hr.

- Price = 22.3¢/Imperial gallon delivered

- Calculation:

$$\frac{51.3 \times .578 \times .75 \times .223}{7.41} = \$ .669$$

- Oil, grease and filters. (15% of fuel cost) \$ .100
- Repairs - .0001 x \$4,300.00 = \$ .430
- Total operating cost per hour \$ 1.199

3.1.4.1.1.3 Operating cost of I.H.C. 300 gasoline:

- Fuel cost

- Data - Maximum drawbar horsepower = 42.8 HP

- Fuel consumption = .639 lb/HP hr.

- Price = 22.3¢/Imperial gallon delivered



- Calculation:

$\frac{42.8 \times .639 \times .75 \times .223}{7.41}$	=	\$ .618
- Oil, grease and filters. (15% of fuel cost)		\$ .092
- Repairs - $.0001 \times \$3,200.00$	=	<u>\$ .320</u>
- Total operating cost per hour		\$ 1.030

3.1.4.1.2 Implement costs:

The usual unit used for cropping activities is the acre. However the costs of operating field equipment are usually given in dollars per hour. Therefore, it is desirable to have costs on a per acre basis and for this the capacity of a machine in acres per hour is required. This may be calculated using the formula from the Agricultural Engineers yearbook<sup>1</sup> which is

$$C = \frac{SWE}{825}$$

where C = actual field capacity in acres/hr.

S = speed in miles per hour

W = rated width of the machine

E = field efficiency in percent.

Speed was selected as 4.5 miles per hour for most field operations unless specified otherwise by the farmer. The rated width was specified by the farmer. The field efficiency was selected according to the table in the yearbook<sup>1</sup>. Implement and Tractor<sup>28</sup> provided combine and baler data for dimensions, capacities, and fuel consumption of auxillary engines. MacHardy's<sup>34</sup> formula for sizing combines was used to obtain the capacity of the combine if the farmer did not specify it. All estimates of repair costs as a percentage of original cost came from the A.S.A.E. yearbook<sup>1</sup>, Table 4.



3.1.4.1.2.1 Cultivator:

- Capacity =  $\frac{5 \times 16 \times 82.5}{825} = 8.0$  acres/hr.
- Operating cost per hour:
  - Tractor (I.H.C. 660) \$ 1.34
  - Repairs to cultivator. (.0006 x \$900.00) \$ .54
  - Total operating cost per hour \$ 1.88
- Operating cost per acre = \$1.88/8.0 = \$ .235

3.1.4.1.2.2 Rodweeder:

- Capacity =  $\frac{5 \times 20 \times 82.5}{825} = 10.0$  acres/hr.
- Operating cost per hour:
  - Tractor (I.H.C. 660) \$ 1.34
  - Repairs to rodweeder. (.0006 x \$1,200.00)  
(Used cultivator rate) \$ .72
  - Total operating cost per hour \$ 2.06
- Operating cost per acre = \$2.06/10.0 = \$ .206

3.1.4.1.2.3 Noble Blade:

- Capacity =  $\frac{5 \times 16 \times 82.5}{825} = 8.0$  acres/hr.
- Operating cost per hour:
  - Tractor (I.H.C. 660) \$ 1.34
  - Repairs to noble blade. (.0006 x \$1,300.00) \$ .78
  - Total operating cost per hour \$ 2.12
- Operating cost per acre = \$2.12/8.0 = \$ .265

3.1.4.1.2.4 Discer:

- Capacity =  $\frac{5 \times 16 \times 60}{825} = 5.8$  acres/hr.  
(seeding)
- Operating cost per hour:
  - Tractor (I.H.C. 660) \$ 1.34
  - Repairs to discer. (.0005 x \$2,000.00)  
(Used one way disc rate) \$ 1.00
  - Total operating cost per hour \$ 2.34
- Operating cost per acre = \$2.34/5.8 = \$ .401



3.1.4.1.2.5 Oscillating harrows:

- Capacity  $= \frac{6 \times 21 \times 82.5}{825} = 12.6$  acres/hr.
- Operating cost per hour:
  - Tractor (I.H.C. 300) \$ 1.03
  - Repairs. (.0004 x \$ 600.00) \$ .24
  - Total operating cost per hour \$ 1.27
- Operating cost per acre  $= \$1.27/12.6 =$  \$ .101

3.1.4.1.2.6 Double disc drill:

- Capacity  $= \frac{5 \times 14 \times 60}{825} = 5.10$  acres/hr.
- Operating cost per hour:
  - Tractor (I.H.C. 300) \$ 1.03
  - Drill repairs. (.0008 x \$1,200.00) \$ .96
  - Total operating cost per hour \$ 1.99
- Operating cost per acre  $= \$1.99/5.10 =$  \$ .391

3.1.4.1.2.7 Single disc plus harrows:

This machine combination is used only for wet springs.

- Capacity  $= \frac{5 \times 20 \times 82.5}{825} = 10.0$  acres/hr.
- Operating cost per hour:
  - Tractor (I.H.C. 450) \$ 1.20
  - Machinery repairs:
    - Harrows (.0004 x \$ 600.00) \$ .24
    - Disc (.00065 x \$ 400.00) \$ .26
  - Total operating cost per hour \$ 1.70
- Operating cost per acre  $= \$1.70/10 =$  \$ .170

3.1.4.1.2.8 Swathers:

This farmer used two p.t.o. swathers running simultaneously.

- Capacity:
  - 15 ft. -  $\frac{5 \times 15 \times 82.5}{825} = 7.5$  acres/hr.
  - 16 ft. -  $\frac{5 \times 16 \times 82.5}{825} = 8.0$  acres/hr.



- Operating cost per hour:	
15 ft. - Tractor (I.H.C. 300)	\$ 1.03
- Swather repairs (.0004 x \$1,550.00)	<u>\$ .62</u>
(Used data for self-propelled unit)	
- Total operating cost per hour	\$ 1.65
16 ft. - Tractor (I.H.C. 450)	\$ 1.10
(Light work, so reduce cost)	
- Swather repairs (.0004 x \$1,600.00)	<u>\$ .64</u>
- Total operating cost per hour	\$ 1.74
- Operating cost per acre	\$ .219
- 15 ft. - \$1.65/7.5 =	\$ .220
- 16 ft. - \$1.74/8.0 =	\$ .218

#### 3.1.4.1.2.9 Combine:

This farmer had two combines which he ran simultaneously. The models were I.H.C. 141 (self-propelled) and I.H.C. 140 (P.T.O.); the farmer felt the capacity of each combine was identical to the other. The capacity on summerfallow crops was the farmer's estimate. The capacity on stubble crops was adjusted slightly higher due to the lower yields.

- Capacity - 4.75 acres/hr. on summerfallow
- 5.00 acres/hr. on stubble

An alternative method (MacHardy<sup>34</sup>, page 62) of calculating combine capacity was used when the farmer did not estimate capacity.

$$Y = 3 \left( \frac{W}{192} + \frac{B^{3/2} \times L}{38,600} + \frac{S}{7,400} \right)$$

Y = field capacity in long tons (2,240 lb/ton) per acre

W = cylinder width in inches

B = body width in inches

L = straw walker length in inches

S = combined chaffer and sieve area in square inches



$$Y = 3\left(\frac{32}{192} + \frac{32^{3/2} \times 122}{38,600} + \frac{2,750}{7,400}\right)$$

$$3(.167 + .572 + .372) = 3.33 \text{ long tons}$$

$$3.33 \times \frac{2240}{2000} = 3.73 \text{ short tons/hr.}$$

- Operating costs per hour:

141 I.H.C.

- Fuel (.25 x 12 x \$ .223)	\$ .669
- Oil and filters. (15% of fuel)	\$ .104
- Repairs. (.00027 x \$6,500.00)	\$ 1.755
- Total operating cost per hour	\$ 2.428

140 I.H.C.

- Tractor (I.H.C. 660)	\$ 1.342
- Repairs. (.00027 x \$3,800.00)	\$ 1.026
- Total operating cost per hour	\$ 2.368

- Operating cost per acre - fallow = $\frac{2.398}{4.75}$	\$ .505
- stubble = $\frac{2.398}{5.00}$	\$ .480

3.1.4.1.2.10 Mower:

$$\text{- Capacity} = \frac{6 \times 7 \times 82.5}{825} = 4.2 \text{ acres/hr.}$$

- Operating cost per hour:

- Tractor cost (I.H.C. 300)	\$ 1.03
- Mower repairs. (.0012 x \$ 600.00)	\$ .72
- Total operating cost per hour	\$ 1.75

- Operating cost per acre	\$ .419
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3.1.4.1.2.11 Rake:

$$\text{- Capacity} = \frac{7 \times 12 \times 82.5}{825} = 8.4 \text{ acres/hr.}$$

- Operating cost per hour:

- Tractor (I.H.C. 450)	\$ 1.20
- Rake repairs. (.0007 x \$ 650.00)	\$ .45
- Total operating cost per hour	\$ 1.65

- Operating cost per acre	\$ .196
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3.1.4.1.2.12 Farmhand Front End Loader:

- Capacity = 25 ton/10 hour day = 2.5 ton/hour
- Operating cost per hour:
  - Tractor (I.H.C. 660) \$ 1.01  
(Light work so cost/hour equals 3/4 of rated power cost)
  - Loader repairs. (.0003 x \$1,600.00) \$ .48
  - Total operating cost per hour \$ 1.49
- Operating cost per ton \$ .596

3.1.4.2 Determining Activity Coefficients.

An outline of the development of matrix coefficients is given in this section. The tables refer to the functional value as the "activity gross margin" because it may not be a true enterprise gross margin. For Cases 1 and 2, the gross margin may be interpreted as return to management and capital. For all other Cases, gross margin is the return to regular labor, management and capital. The charge for casual labor was \$1.80 per hour for Cases 1 and 2 and \$1.00 per hour for all other Cases. In interpreting Cases 1 and 2, it should be recognized that the solution reflects the higher charges for casual labor.

3.1.4.2.1 Crops:

This includes activities 1 to 13 which are based on the unit of one acre of land. Summaries of the information used for each crop are given in Tables 4 to 15 inclusive.



Table 4. Data for Hard Red Spring Wheat Seeded on Fallow  
(Activity 1)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seed	\$ 2.40	.			
Seeding with Discer	\$ .401	.086	Apr.	23	.086
		.086	May		
Harrowing	\$ .101	.079	May	24	.165
Spraying	\$ .70				
Swathing	\$ .219	.133	Aug.	27	.133
Combining (2 men required)	\$ .505	.424	Sept.	28	.424
Labor cost (.17 + .08 + .13 + .21) x \$ .90	\$ .531				
Total operating cost per acre	\$ 4.857				
Gross returns (21 bus/acre x \$1.50/bu)	\$31.50				
Activity gross margin (\$31.50 - \$ 4.85*)	\$26.65				
Acreage requirements:					
- One acre of arable land				1	1.
- One acre of fallow land to seed				4	1.
Pasture and Straw Yield:					
- Winter pasture capacity equivalent to .2 acre native winter range				8	-.2
- .5 ton of straw to be picked up if required				32	-.5

\* This figure should be rounded to \$4.86 rather than \$4.85.



Table 5. Data for Flax Seeded on Fallow  
(Activity 2)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seed	\$ 4.00				
Fertilizer, 11-48-0 @ 40 lb/ac. (\$105.00/ton x $\frac{40}{2000}$ ton)	\$ 2.20				
Machine and labor cost (Farmer's machinery only which included discer, harrows, swathers and combine.)	\$ 1.75*	.086	Apr.	23	.086
		.165	May	24	.165
		.133	Aug.	27	.133
		.424	Sept.	28	.424
Spraying	\$ .90				
Total operating cost per acre	\$ 8.85				
Gross returns (15 bus/acre x \$3.00/bu)	\$45.00				
Activity gross margin (\$45.00 - \$ 8.85)	\$36.15				
Acreage requirements:					
- One acre of arable land				1	1.
- One acre of flax acreage limit to seed				2	1.
- One acre of fallow land to seed				4	1.

\* This figure should be rounded to \$1.76 instead of \$1.75.



Table 6. Data for Oats Seeded on Fallow  
(Activity 3)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seed	\$ 1.50				
Fertilizer, 11-48-0 @ 40 lb/ac.	\$ 2.20				
Machine and labor cost (Farmer's machinery only)	\$ 1.75*	.251	May	24	.251
		.133	Aug.	27	.133
		.424	Sept.	28	.424
Spraying	\$ .70				
Total operating cost per acre	\$ 6.15				
Activity gross margin	-\$ 6.15				
Acreage requirements:					
- One acre of arable land				1	1.
- One acre of fallow to seed				4	1.
Grain, pasture and straw yield:					
- 44 bushels of oats for sale or feed				11	-44.
- Winter pasture capacity equivalent to .2 acre of native winter range				8	-.2
- .8 ton of straw to be picked up if required				32	-.8

\* This figure should be rounded to \$1.76 instead of \$1.75.



Table 7. Data for Barley Seeded on Fallow  
(Activity 4)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seed	\$ 1.75				
Machine and labor cost (Farmer's machinery only)	\$ 1.75*	.251	May	24	.251
		.133	Aug.	27	.133
		.424	Sept.	28	.424
Spraying	\$ .70				
Total operating cost per acre	\$ 4.20				
Activity gross margin	\$ 4.20				
Acreage requirements:					
- One acre of arable land				1	1.
- One acre of fallow to seed				4	1.
Pasture and grain yields:					
- Winter pasture capacity equivalent to .2 acre of native winter range				8	-.2
- 30 bushels of barley for sale or feed				12	-.30

\* This figure should have been rounded to \$1.76 instead of \$1.75.



Table 8. Data for Durum Wheat Seeded on Fallow  
(Activity 5)

Item	Dollars Per Acre	Labor			
		Required Hr.	Mo.	Row Number	Matrix Coefficient
Seed	\$ 3.00				
Fertilizer, 16-48-0 @ 45 lb/ac. ( $\$110.00/\text{ton} \times \frac{45}{2000} \text{ ton}$ )	\$ 2.50*				
Machine and labor cost (Farmer's machinery only)	\$ 1.75**	.086	Apr.	23	.086
		.165	May	24	.165
		.133	Aug.	27	.133
		.424	Sept.	28	.424
Spraying	\$ .70				
Total operating cost per acre	\$ 7.95				
Gross margin (21 bus/acre x \$1.75/bu)	\$36.75				
Activity gross margin (36.75 - 7.95)	\$28.80				
Acreage requirements:					
- One acre of arable land				1	1.
- One acre of fallow to seed				4	1.
Winter pasture yield:					
- Winter pasture equivalent to .2 acre native winter range				8	-.2

\* This was rounded off to \$2.50 when it should be \$2.48.

\*\* This figure was rounded off to \$1.75 when it should have been \$1.76.



Table 9. Data for Hard Red Spring Wheat Seeded on Stubble  
(Activity 6)

Item	Dollars Per Acre	Labor Required Hr. Mo.	Row Number	Matrix Coefficient
Seed	\$ 2.40			
Cultivate	\$ .235	.125 Apr.	23	.125
Seeding with drill	\$ .391	.172 May		
Harrows	\$ .101	.079 May	24	.275
Spraying	\$ .70			
Swathing	\$ .219	.133 Aug.	27	.133
Combining (2 men required)	\$ .480	.400 Sept.	28	.400
Labor charge (.12 + .17 + .08 + .13 + .20) x \$ .90	\$ .630			
Total operating costs per acre	\$ 5.156			
Gross returns (14.5 bus/acre x \$1.50/bu)	\$21.75			
Activity gross margin (31.75 - 5.15*)	\$16.60			
**Acreage requirements:				
- One acre of arable land			1	1.
Pasture and straw yields:				
- Winter pasture capacity equivalent to .2 acre native winter range			8	-.2
- .35 ton of straw to be picked up if required			32	-.35

\* This was rounded to \$5.15 when it should be \$5.16.

\*\* A stubble balance equation was not placed in the program since the higher returns of grain on fallow meant that the stubble crops would be selected after the fallow was filled.



Table 10. Data for Oats Seeded on Stubble  
(Activity 7)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seed	\$ 1.50				
Fertilizer* ( $\frac{65}{2000}$ ton x \$105.00/ton)	\$ 3.40**				
Cultivate	\$ 2.235	.125	Apr.	23	.125
Seeding with discer	\$ .401	.172	May	24	.251
Harrows	\$ .101	.079	May		
Spraying	\$ .70				
Swathing	\$ .219	.133	Aug.	27	.133
Combining (2 men required)	\$ .480	.400	Sept.	28	.400
Labor (.12 + .17 + .08 + .13 + .20) x \$ .90	\$ .630				
Total operating cost per acre	\$ 7.666				
Activity gross margin	\$-7.65***				
Acreage requirement****:					
- One acre of arable land				1	1.
Grain, pasture, and straw yield:					
- 30 bus. of oats for sale or feed				11	-30.
- Winter pasture equivalent to .2 acre native winter range				8	-.2
- .6 ton of straw to be picked up if required				32	-.6

\* 11-48-0 was used instead of 16-48-0 as the farmer wanted.

\*\* This was rounded to \$3.40 when it should be \$3.413.

\*\*\* This figure was rounded to \$7.65 when it should be \$7.67.

\*\*\*\* A stubble balance was not required.



Table 11. Data for Barley Seeded on Stubble  
(Activity 8)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seed	\$ 1.75				
Machine and labor cost (Farmer's machinery only)	\$ 2.05*	.125	Apr.	23	.125
		.251	May	24	.251
		.133	Aug.	27	.133
		.400	Sept.	28	.400
Spraying	\$ .70				
Total cost per acre	\$ 4.50				
Activity gross margin	\$-4.50				
Acreage requirement:					
- One acre of arable land				1	1.
Grain and pasture yields:					
- 20 bushels of barley for sale or feed				12	-20.
- Winter pasture capacity equivalent to .2 acre native winter range				8	-.2

\* This was rounded off to \$2.05 when it should be \$2.067.



Table 12. Data for Land Summerfallowed  
(Activity 9)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Noble Blade	\$ .265	.125	May	24	.125
Cultivate with Rodweeder Attachment (Added 15% to compensate for the attachment)	\$ .270	.125	June	25	.125
Rodweed	\$ .206	.100	July	26	.100
Labor cost (.125 + .125 + .100) x \$ .90	\$ .315				
Total operating cost per acre	\$ 1.056				
Activity gross margin	\$-1.06				
Acreage requirements:					
- One acre of arable land				1	1.
- One acre of the minimum fallow requirement				3	1.
Land made available:					
- One acre of fallow land for seeding				4	-1.
- One acre for seeding to cover crop in year it is fallow				5	-1.



Cover crop has relatively widespread application in southern Alberta due to the long fall grazing season. It consists of oats seeded in late July on land being summerfallowed in the current season in order to provide lush pasture in the late fall. The penalty for seeding the crop was estimated by the farmer to be a reduced yield of the following crop equivalent to 5 bu./acre of wheat. To allow for the lower value of coarse grains, this grain was assumed to be worth \$1.40/bu. The farmer felt the only economic advantage of the crop was in the nutrients produced. Other advantages are often mentioned such as protection of the soil from wind erosion and more complete land use.

The following procedure was used to convert the farmer's estimate to pounds of nutrient per acre.

- Total weight of pasture produced:

- Dry matter: 2/3 yearling eating 20 lb. dry matter/day  
for 30 days.

$$2/3 \times 20 \times 30 = 400 \text{ lb./acre}$$

- Wet weight: Morrison<sup>38</sup> lists oat pasture at 14% dry matter.

$$\frac{400}{.14} = 2,877 \text{ lb./acre}$$

- Nutrients (Morrison's<sup>38</sup> analysis):

$$\begin{array}{l} \text{- TDN: } \frac{.097 \text{ lb. TDN}}{\text{lb. feed}} \times \frac{2,857 \text{ lb. feed}}{\text{acre}} = 278 \text{ lb. TDN/acre} \\ \text{- DP : } \frac{.024 \text{ lb. DP}}{\text{lb. feed}} \times \frac{2,857 \text{ lb. feed}}{\text{acre}} = 68 \text{ lb. DP/acre} \end{array}$$

Since the crop was seeded in July, it was assumed that only regular labor was required. Therefore labor costs were not considered in the activity costs. Table 13 summarizes the data for cover crop which is actually two activities. The only difference lies in the use of TDN.



Table 13. Data Used for Cover Crop  
(Activity 10 and Activity 11)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Cultivate	\$ .235	.125	May	24	.125
Seeding with Discer	\$ .401	.172	July	26	.251
Harrows	\$ .101	.079			
Seed (oats)	\$ 1.10				
Yield loss (5 bu/acre x \$1.40/bu)	\$ 7.00				
Total operating cost per acre	\$ 8.837				
Activity gross margin	\$-8.83*				
Acreage requirements:					
- One acre of land being left fallow in the current season				5	1.
- Animals available to graze at least one acre				6	1.
Nutrient yield:					
- 278 pounds of total digestible nutrients which may be used for maintenance or production				15	-278.
				(Activity 10 only)	
				16	-278.
				(Activity 11 only)	
- 68 lb. of digestible protein				17	-68.

\* This figure should be rounded off to \$8.84.



Table 14. Data for Tame Hay\*  
(Activity 12)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seeding .2 acres					
Seed (.2 x 3.50)	\$ .700				
Seeding (.2 x .401)	\$ .080	.034	May	24	.066
Harrow twice (.4 x .101)	\$ .040	.032	May		
Mow 1 acre	\$ .419	.242	June	25	.288
Rake 1 acre	\$ .196	.046	June		
Stack		.073	July	26	.288
(.5 ton/acre x \$ .596/ton)	\$ .298	.200	July		
Cultivate 3 times (.6 x .235)	\$ .141	.015	July		
		.020	Aug.	27	.020
		.020	Sept.	28	.020
		.020	Oct.	29	.020
Labor cost (.702 x \$ .90)	\$ .632				
Total operating cost per acre	\$ 2.506				
Activity gross margin	\$-2.51				
Acreage requirement:					
- One acre of arable land				1	1.
- 1/3 acre of fallow land**				3	.33
Hay yield:					
- .5 ton of grass hay for sale or feed				13	-.5

\* Reseeding costs were based on 20% of the land being broken up after July 15 for reseeding the following year. The procedure was cultivate three times in the fall and harrow, seed with the discer, and harrow the following May. Haying was done in June and early July.

\*\* Each acre in hay fills one third of an acre of the minimum summerfallow restriction since the farmer felt that land seeded to grass did not require summerfallow.



Table 15. Data for Tame Pasture\*  
(Activity 13)

Item	Dollars Per Acre	Labor Required		Row Number	Matrix Coefficient
		Hr.	Mo.		
Seed .2 acre	\$ .70				
Seeding .2 acre	\$ .080	.034	May	24	.066
Harrowing .4 acre	\$ .040	.032	May		
Cultivate .6 acre	\$ .141	.015	July	26	.015
		.020	Aug.	27	.020
		.020	Sept.	28	.020
		.020	Oct.	29	.020
Total operating cost per acre	\$ .961				
Activity gross margin	\$ -.96				
Acreage requirement:					
- One acre of arable land				1	1.
- 1/3 acre of minimum requirement on summerfallow				3	.33
Pasture yield:					
- Pasture capacity equivalent to 3 acres native range				7	-3.

\* This activity has the same reseeding practice as tame hay. It was assumed that regular labor did all the work for this activity.



### 3.1.4.2.2 Transfer activities:

These are activities which permit adjustment in the supplies of resources allocated to different areas of the farm. Both such activities in Case 1 are based on one acre units and are as follows:

#### a) Excess summerfallow (Activity 14).

This activity is actually a slack variable to make the summerfallow restriction an equality and allows the amount of fallow to exceed the lower limit in the program. It need not be here but can be added at the end of the program. The only entry is a -1. in the third row.

#### b) Winter grass (Activity 15).

This allows the saving of grass for winter pasture rather than allowing all the pasture to be used for summer grazing. The only entries are +1. in row 7 and -1. in row 8.

### 3.1.4.2.3 Supplies purchased:

Table 16. A Summary of the Data Used for Purchased Supplies

Activity			Cost		Activity		Resource Supplied	
Name	Number	Unit	Per Unit		Gross Margin	Row	Matrix	Coeff.
Rent winter pasture	16	Acre	\$ .60*		\$ -.60	8		-1.
Buy Straw	17	Ton	\$ 8.00		\$ -8.00	14		-1.
Buy Oats	18	Bushe1	\$ .55		\$ -.55	11		-1.
Buy Hay	19	Ton	\$20.00		\$-20.00	13		-1.
Buy Barley	20	Bushe1	\$ .80		\$ -.80	12		-1.

\* Basis: Pasture rent = \$9.00/winter/cow. One cow requires 15 acres native range or equivalent for 6 months winter pasture of which the last 3 months usually require supplemental feed.



These activities were primarily livestock feeds which could be purchased to add to the farm supply of that resource. Table 16 summarizes the data used in Case 1.

#### 3.1.4.2.4 Winter feeds:

The feed analysis obtained from Morrison<sup>38</sup>, Table 1 was discussed with Dr. J.P. Bowland<sup>10</sup> to determine any differences in the feeding value of the feeds when used for maintenance as opposed to fattening. He felt that the only differences would be in oats and winter pasture. On this basis, oats used for maintenance were assumed to supply the same energy as barley and winter pasture could be used only for maintenance of the cow herd.

There are some calculations required to convert the farmer's data to usable figures. These calculations are:

- Winter pasture:

- Air dry feed: One cow eating 28 lb./day for 100 days requires 8 acres of native range.

$$\frac{28 \times 100}{8} = 350 \text{ lb./acre}$$

- Pellets:

- Composition: 1/3 dehydrated alfalfa, 1/3 dehydrated beet pulp, and 1/3 screenings.

- Cost: \$60.00 per ton (\$3.00 per 100 lb.)

- Analysis:

- Total digestible nutrients:

$$\text{Alfalfa: } 1/3 \times 100 \times .536 = 17.9$$

$$\text{Beet pulp: } 1/3 \times 100 \times .687 = 22.9$$

$$\text{Screenings: } 1/3 \times 100 \times .628 = \underline{20.9}$$

$$\text{TDN/100 lb.} = 61.7 \text{ lb.}$$



Table 17. Nutrients Supplied for Winter Feeding by the Feeds Available

Feed	Unit	Activity Number	Nutrients Supplied				Other Entries	
			TDN (1b) (Row 15)	TDN (1b) (Prod.) (Row 16)	Digestible Protein (1b) (Row 17)	Matrix Coeff.	Requirement For	
							Row	
Winter Pasture	Acre	21	168 (.43 x 350)		12 (.035 x 350)	1.	8	Winter pasture
Oats for Maintenance	Bushel	22	26.2 (.77 x 34)		3.2 (.095 x 34)	1.	9	Enough cows to pasture 1 acre
Oats for Production	Bushel	23		23.8 (.70 x 34)	3.2 (.095 x 34)	1.	11	Oats
Barley for Maintenance	Bushel	24	36.9 (.77 x 48)		4.8 (.10 x 48)	1.	12	Barley
Barley for Production	Bushel	25		36.9 (.77 x 48)	4.8 (.10 x 48)	1.	12	Barley
Hay Maintenance	Ton	26	902 (.451 x 2000)		40 (.020 x 2000)	1.	13	Hay
Straw Maintenance	Ton	27	896 (.448 x 2000)		14 (.007 x 2000)	1.	14	Straw
32% Supplement Maintenance	100 lb.	28	75 (.75 x 100)		30 (.30 x 100)	- \$4.50	A.G.	
32% Supplement Production	100 lb.	29		75 (.75 x 100)	30 (.30 x 100)	- \$4.50	A.G.	
Pellets for Maintenance	100 lb.	30	61.7		7.7	- \$3.00	A.G.	
Pellets for Production	100 lb.	31		61.7	7.7	- \$3.00	A.G.	



- Digestible protein:

$$\text{Alfalfa: } 1/3 \times 100 \times .125 = 3.2$$

$$\text{Beet pulp: } 1/3 \times 100 \times .041 = 1.4$$

$$\text{Screenings: } 1/3 \times 100 \times .092 = \underline{3.1}$$

$$\text{DP/100 lb.} = 7.7 \text{ lb.}$$

Table 17 summarizes the coefficients used in Case 1. In the table, A.G. refers to the "activity gross margin" and the negative signs for the matrix coefficients have not been entered under the "nutrients supplied" heading.

3.1.4.2.5 Summer Feeds:

The analysis for these is the same as for winter feeds, but the entries of the nutrients supplied are on the nutrient balance rows for summer (rows 18, 19, and 20) instead of winter (rows 15, 16, and 17). Straw was left out of the group and summer pasture was added on the following basis:

- Grass yield:

One cow (nursing a calf) eating 85 lb. per day (wet basis) for 185 days requires 15,725 lb. of feed (wet basis) from 25 acres of native range. One acre of native range supplies 628 lbs. feed/acre (wet basis).

- Nutrients supplied:

Morrison's<sup>38</sup> analysis shows grass will contain 25.4% TDN and 1.6% DP which gives 160 lbs. TDN per acre and 10 lb. DP per acre if used for maintenance. If the pasture is used for



fattening animals (production), then only 75% of the TDN is usable so the available TDN is 120 lb. per acre.

#### 3.1.4.2.6 Livestock:

The basis of the nutrient requirements was NRC<sup>40, 41</sup> data. This was recommended by Dr. Bowland<sup>10</sup> in personal discussion. However, he felt that Morrison<sup>38</sup> had satisfactory average analysis figures of feeds for ruminant animals but to use other sources for non-ruminant animals. All activities are based on one animal of the class being one unit. Also returns per unit are returns to feed, management, and capital. The activity gross margin is the returns after the labor and marketing costs have been deducted.

##### 3.1.4.2.6.1 Cows (Activity 32):

- Maximum cover crop each cow may utilize is 5 acres. (Row 6)
- Each cow requires 25 acres native range for summer pasture. (Row 7)
- Cows used - each cow must be supplied by those on hand or by purchase. (Row 10)

- Bedding required (straw). (Row 14)

$$\frac{2.25 \text{ lb/day} \times 180 \text{ days}}{2,000 \text{ lb/ton}} = .2 \text{ ton}$$

- Nutrient requirements. (Rows 15 and 17)

TDN (maintenance only) (10.0 x 180) 1,800 lb.

DP (.8 x 180) 144 lb.

- Labor. (Rows 23 through 30)

- The farmer estimated the beef cow operation required 6.4 months to care for 133 cows. This was assumed to



include the moving, fencing, and periodic inspection which remained relatively stable regardless of numbers. Thus it was estimated that  $\frac{2}{3}$  of this time or 4.3 months was time which actually depended on the number of cows on hand. As well, it was estimated that 80% of the 4.3 months was winter feeding time when the work per month per man was 150 hours. The other 20% of the time was months when the time per man per month was 200 hours. Therefore the total time for 133 cows was:

$$.80 \times 4.3 \times 150 = 516 \text{ hours}$$

$$.20 \times 4.3 \times 200 = \underline{168 \text{ hours}}$$

$$\text{Total} = 684 \text{ hours}$$

$$\text{Time per cow} = \frac{684}{133} = 5.15 \text{ hours}$$

This was broken down as follows:

October - .50 hours extra time for weaning and moving.

Winter - 4.15 hours primarily winter feeding.

April - .50 hours extra time for calving and branding.

- Selling cost per calf:

- Hauling charges (\$60.00/50 calves)	\$ 1.20
- Selling (yardage and commission etc.)	<u>\$ 2.00</u>
	\$ 3.20

- Returns per calf:

- Gross (\$ .26/lb x 450 lb.)	\$117.00
- After selling charges removed	\$113.80

- Returns per cow (90% calf crop) \$102.42



- Cow costs:

- Veterinary fees	\$ 1.00
- Minerals, Vitamin A	\$ 1.00
- Labor (5.15 x \$ .90)	<u>\$ 4.64</u>
	\$ 6.64
- Activity gross margin (102.42 - 6.64)	\$ 95.80

3.1.4.2.6.2 Fatten calves (Activity 33):

- Each calf may consume a maximum of 1.5 acres of cover crop.  
(Row 6)

- Bedding =  $\frac{7 \text{ lb./day} \times 210 \text{ days}}{2,000 \text{ lb./ton}} = .75 \text{ ton (Row 14)}$

- Feeding period 210 days

- Nutrient requirements:

This is based on daily allowance of nutrients which is the average amount per day required. Thus,  $\frac{1}{2}$  the daily requirement of a 450 lb. calf and  $\frac{1}{2}$  the daily requirement of an 850 lb. calf gives the average daily requirement for fattening calves. Maintenance is determined from the corresponding wintering requirement (refer to wintering calves).

- Daily requirement:

TDN 11.6 lb./day ( $\frac{1}{2} \times 9.0 + \frac{1}{2} \times 14.2$ )

DP 1.44 lb./day ( $\frac{1}{2} \times 1.23 + \frac{1}{2} \times 1.65$ )

- Maintenance requirement TDN 6.6 lb./day (see wintering calves)
- Production requirement TDN 5.0 lb./day



- Total nutrient requirement:

$6.6 \times 210 = 1,396$  lb. TDN for maintenance. (Row 15)

$5.0 \times 210 = 1,050$  lb. TDN for production. (Row 16)

$1.44 \times 210 = 302$  lb. DP. (Row 17)

- Labor. (Rows 23 through 30)

- The farmer estimated 3.2 months were required to fatten 98 head of calves. This was assumed to be based on 150 hours per man per month.

- Time:

Total =  $3.2 \times 150 = 480$  hours

Per calf =  $\frac{480}{98} = 4.90$  hours

- This operator followed the practice of feeding a great deal of roughage for the first 4 months or 120 days from December 1 to late March, which meant his calves were growing but not fattening. Then he put them on grain until they were on self feed by mid April. Thus the time was classified as:

Winter 4.00 hours

April .60 hours

May .15 hours

June .15 hours

- Feedlot requirement:

- Space equivalent to .75 yearling steers. (Row 31)



- Partial Budget:

- Purchase at 450 lb. for \$ .26/lb. (Row 22)	\$117.00
- Cost:	
- Buy - trucking	\$ .60
- commission	\$ .68
- Sell - trucking	\$ 2.00
- commission	\$ 1.50
- yardage	\$ 1.25
- Losses 2%	\$ 2.34
- Veterinary, minerals, Vitamin A	\$ 3.85
- Labor (4.90 x \$ .90)	<u>\$ 4.41</u>
	\$ 16.63
- Gross returns (850 lb. for \$ .2325/lb.)	\$197.63
- Activity gross margin (197.63 - 117.00 - 16.63)	\$ 64.00

3.1.4.2.6.3 Winter calves (Activity 34):

- Maximum cover crop consumption 2.0 acres (Row 6)
- Bedding - same as fattening calves .75 ton (Row 14)
- Nutrient requirements (180 day feeding period):  
  
    Basis:  $\frac{1}{2}$  requirement for 450 lb. calf +  $\frac{1}{2}$  requirement  
            for 600 lb. calf.  
  
    TDN maintenance (6.6 x 180) 1,188 lb. (Row 15)  
  
    DP (.78 x 180) 140 lb. (Row 17)
- Labor - 60 calves require 1 hour per day to care for.
  - $\frac{1}{60} \times 180 = 3.0$  hr/head of winter labor for the  
    period. (Row 30)



- Partial budget:

- Purchase at 450 lb. for \$ .26/lb. (Row 22)	\$117.00
- Cost:	
- Buy - freight	\$ .60
- commission	\$ .68
- Sell - freight	\$ 1.50
- commission	\$ 1.50
- yardage	\$ 1.26
- Losses 1%	\$ 1.17
- Veterinary fees, minerals, Vitamin A	\$ 1.59
- Labor (3.00 x \$ .90)	<u>\$ 2.70</u>
	\$ 11.00
- Gross returns (600 lb. for \$ .24/lb.)	\$144.00
- Activity gross margin (144.00 - 117.00 - 11.00)	\$ 16.00

3.1.4.2.6.4 Pasture yearlings (Activity 35):

- Pasture period = 180 days.
- Pasture requirements (75% of cow requirement)
  - .75 x 25 = 17 acre native range. (Row 7)
- Labor:
  - 60 calves to buy and haul to pasture or to round up and ship requires 10 hours.
  - Checking requires 6 hours.
  - Time/yearling:  $\frac{10}{60} = .167$  hours late April. (Row 23)
    - plus  $\frac{10}{60} = .167$  hours early October. (Row 29)
    - plus  $\frac{6}{60} = .10$  hour in July for checking. (Row 26)



- Partial budget:

- Purchase at 600 lb. for \$ .24/lb. (Row 22)	\$144.00
- Cost:	
- Buy - freight	\$ .60
- commission	\$ .90
- Sell - freight	\$ 2.00
- commission	\$ 1.20
- yardage	\$ 1.26
- Veterinary fees, minerals	\$ .65
- Labor (.43 x .9)	<u>\$ .39</u>
	\$ 7.00
- Gross returns (800 lb. for \$ .22/lb)	\$176.00
- Activity gross margin (176.00 - 144.00 - 7.00)	\$ 25.00

3.1.4.2.6.5 Fatten yearlings over summer (Activity 36):

- Bedding =  $\frac{6 \text{ lb./day} \times 180 \text{ days}}{2,000 \text{ lb./ton}} = .5 \text{ ton (Row 14)}$

- Feeding period 180 days.

- Nutrient requirements:

TDN maintenance (8.5 x 180) 1,530 lb. (Row 18)

TDN production (6.5 x 180) 1,170 lb. (Row 19)

Digestible protein (1.62 x 180) 292 lb. (Row 20)

- Feedlot space - space for one yearling in summer. (Row 33)

- Labor:

- The yearlings are self fed for the last three months

so time is the same as fattening calves.

- Time for August, September and October = .15/hr./head/month.  
(Rows 27, 28, and 29)



- Hand feeding time is based on the ratio of the amount of feed consumed by calves (18 lb./day total) to that consumed by yearlings (27 lb./day total). Thus yearlings require 1.5 times as much time for hand feeding.

- Time: May, June, and July = 1.5 hr./head/month.  
(Rows 24, 25, and 26)

- Vaccinate, buy, haul, implant etc. in late April = 2.00 hr/head

- Partial budget:

- Purchase at 600 lb. for \$ .24/lb. (Row 22) \$144.00

- Cost:

- Buy - freight \$ .60

- commission \$ .90

- Sell - freight \$ 2.20

- commission \$ 1.43

- yardage \$ 1.26

- Losses 1% \$ 1.44

- Veterinary fees, minerals, Vitamin A \$ 1.92

- Labor (6.95 x .9) \$ 6.25

\$ 16.00

- Gross returns (950 lb. for \$ .24/lb) \$228.00

- Activity gross margin  
(228.00 - 144.00 - 16.00) \$ 68.00

3.1.4.2.6.6 Fatten long yearlings in winter (Activity 37):

- Maximum cover crop consumption- 2.25 acres (Row 6)

- Bedding  $\frac{7 \text{ lb./day} \times 150 \text{ days}}{2,000 \text{ lb./ton}} = .6 \text{ ton}$  (Row 14)

- Feeding period- 150 days.



- Nutrient requirements:

TDN maintenance (10 x 150) 1,500 lb. (Row 15)

TDN production (8 x 150) 1,200 lb. (Row 16)

Digestible protein (1.8 x 150) 270 lb. (Row 17)

- Labor:

- This operation required 1.5 months of hand feeding and 3.5 months self feeding. Self feeding required .20 hr/head/month due to more feed handled than for yearlings or calves.

- To put on self feed, 1.54 hr/head/month were required.  
(Basis: one hour/day for 20 head)

- Total time (1.5 x 1.54 + 3.5 x .2) = 3.0 hours winter labor (Row 30)

- Feedlot space - space for one yearling in winter. (Row 31)

- Partial budget:

- Purchase at 800 lb. for \$ .22/lb. (Row 22) \$176.00

- Cost:

- Buy - freight \$ 1.20

- commission \$ 1.20

- Sell - freight \$ 2.40

- commission \$ 1.65

- yardage \$ 1.26

- Losses 2% \$ 3.51

- Veterinary fees, minerals, Vitamin A \$ 4.67

- Labor (.90 x 3.00) \$ 2.70

\$ 18.70

- Gross returns (1,100 lb. for \$ .227/lb) \$247.70

- Activity gross margin  
(247.70 - 176.00 - 18.70) \$ 55.00



### 3.1.4.2.7 Altering physical plant size:

#### 3.1.4.2.7.1 Buy or sell cows (Activities 49 and 50):

- Unit - one cow
- Value of cow \$165.00 (Row 22)

The matrix coefficient is -165. if a cow is sold (Activity 50), and +165. if a cow is purchased (Activity 49).

- Cow purchases are charged \$10.00 (activity gross margin is -10) for interest and buying cost while cows sold have no value for the activity gross margin.

- The exchange adds (activity 49) or subtracts (activity 50) one cow from the herd. (Row 10)

#### 3.1.4.2.7.2 Buy land:

- Arable land. (Activity 51)
  - Unit - one acre
  - This adds one acre to the arable land (Row 1) and 1/3 of an acre to the minimum summerfallow requirement. (Row 3)
  - Cost - \$85.00/acre capital outlay (Row 21)
    - \$ 4.25/acre/year interest (activity gross margin -4.25)
- Pasture land. (Activity 52)
  - Unit - one acre
  - This adds one acre to the pasture land (Row 7)
  - Cost - \$40.00/acre capital outlay (Row 21)
    - \$ 2.00/acre/year interest (activity gross margin -2.)



3.1.4.2.7.3 Feedlot space (Activity 53):

- Unit - space for one yearling steer
- Farmer's estimated cost: \$900.00 to add capacity of 150 yearling steers to lot or - \$6.00/head capital outlay (Row 21)
  - \$ .30/head/year interest (activity gross margin - .3)
- This adds one unit of capacity to the feedlot for both summer feeding (Row 33) and for winter feeding (row 31)

3.1.4.2.8 Labor hired (Activities 54 through 61):

- Unit - one hour
- These activities were added to permit casual labor to be hired at peak seasons.
- Casual labor costs were \$ .90/hour more than regular labor so the activity gross margin remains -.90 for all activities. A value of -1. is entered in the correct column and row to add the hour of labor to the appropriate supply.

3.1.4.2.9 Sell crops:

These activities resulted from their existing alternative uses for the end product of some previous activities. Table 16a gives a summary of the data.

Table 16a. Data Used for Selling Activities

Activity			Dollars	Activity	Resource Requirement	
Name	Number	Unit	Per Unit	Gross Margin	Row No.	Matrix Coeff.
Sell Oats	62	Bushel	\$ .55	-\$ .55	11	1.
Sell Barley	63	Bushel	\$ .80	-\$ .80	12	1.
Sell Hay	64	Ton	\$20.00	-\$20.00	13	1.



3.1.4.2.10 Piling straw (Activity 65):

This is a transfer column which allows the alternative of having the straw on the land or using it at a different price than buying.

- Unit - one ton

- Supply - There must be enough land in grain to enable the quantity of straw to be piled (Row 32)

- Labor - Piling straw with front end loader.

- Rate: 7 tons/hour = .145 hours/ton

- Cost - machine - \$ .60/ton

- fertilizer loss\* - \$5.25/ton

- labor (.90 x 1.45)- \$ .13/ton

\$5.98/ton

- Activity gross margin -6.0.

\* The calculation was based on the elemental analysis of wheat straw from C.F. Bentley's<sup>7</sup> notes; 12 lb. N/ton, 2 lb.  $P_2O_5$ /ton, 9 lb.  $K_2O$ /ton.

11-48-0 costs \$5.25/100 lb., which supplies approximately equal amounts of nitrogen and more phosphorus than in one ton of straw. Thus one ton of straw was judged to remove nutrients of approximately the value of 100 lb. of 11-48-0. A better cost figure would be to use those given by Bentley<sup>7</sup> which would bring the fertilizer loss to \$2.73/ton and the cost of picking up straw to \$3.46/ton. However, the farmers generally ignored the fertilizer loss so even this cost could have been left out.



Item	Quantity	Unit	Value	Cost	Profit	Net	Gain	Loss	Total
Buy Barley	20	Bushel	1.80						1.80
Winter Pasture (Maintenance)	21	Acres		1.12					1.12
Oats (Maintenance)	22	Bushel		1.26					1.26
Oats (Production)	23	Bushel		1.23					1.23
Barley (Maintenance)	24	Bushel		1.36					1.36
Barley (Production)	25	Bushel		1.48					1.48
Hay (Maintenance)	26	Ton		1.40					1.40
Straw (Maintenance)	27	Ton		1.14					1.14
Supplement (Maintenance)	28	100 lb.		1.30					1.30
Supplement (Production)	29	100 lb.		1.30					1.30
Pellets (Maintenance)	30	100 lb.		1.71					1.71
Pellets (Production)	31	100 lb.		1.50					1.50
Cow-calf (sell calf in fall)	32	Cow		1.75					1.75
Fatten Calves	33	Calf		1.60					1.60
Victor Calves	34	Calf		1.40					1.40
Summer Pasture Yearlings	35	Yearling		1.25					1.25
Spring Yearlings (Summer)	36	Yearling		1.50					1.50
Fatten Yearlings (Winter)	37	Yearling		1.70					1.70
Summer Pasture (Maintenance)	38	Acres		1.10					1.10
Summer Pasture (Production)	39	Acres		1.20					1.20
Oats (Maintenance)	40	Bushel		1.26					1.26
Oats (Production)	41	Bushel		1.38					1.38
Barley (Maintenance)	42	Bushel		1.36					1.36
Barley (Production)	43	Bushel		1.48					1.48
Hay (Maintenance)	44	Ton		1.40					1.40
Pellets (Maintenance)	45	100 lb.		1.30					1.30
Pellets (Production)	46	100 lb.		1.30					1.30
Supplement (Maintenance)	47	100 lb.		1.30					1.30
Supplement (Production)	48	100 lb.		1.30					1.30
Buy Cows	49	Cow		1.16					1.16
Sell Cows	50	Cow		1.16					1.16
Buy Arable Land	51	Acres		1.35					1.35



3.1.5 Results.

Gross Margin - \$56,000.00

Cropping Program

Purchase 1,000 acres arable land.

Flax - 100 acres

Durum Wheat on fallow - 603 acres

Fallow - 703 acres

Spring wheat (stubble) - 1178 acres

Livestock Program

Purchase 109 cows to bring herd to 239.

Fatten 150 calves and sell 75.

Feed (winter)

- Winter pasture - 2,400 acres native range equivalent  
(Home stubble land plus rented land)

- Straw - 237 tons.

- Cover crop - 703 acres.

- Barley - 75 bushels (purchased)

Miscellaneous

Hire casual labor equivalent to one man working from April to October plus an additional man in September.

3.1.6 Remarks.

There have been cases where the rounding off of the numbers in the third and fourth figure may have slightly changed the solution. These have been noted in the text. As well, there are several programming assumptions



which may be questioned. These assumptions were that

- a) Fattening stock need not be separated as to sex of the animal.
- b) No upper limit was required on the amount of additional land that could have been handled with the present equipment.
- c) No restriction was required on the roughage intake of fattening animals.
- d) A four year rotation could be specified instead of the three year rotation the farmer requested.
- e) The replacement of the cow herd need not be accounted for by depreciating the cow or by adjusting pasture and feeding requirements. Also the cost of bulls was left out.
- f) Operating capital would not be limited.
- g) The buying and selling price of feeds such as barley, oats or hay need not be separated.

Work to prepare the program began on October 15, 1963 and required until December 18, 1963 to complete. This includes the time required to become familiar with the requirements of the computer program written by Wood<sup>52</sup>.

The data was punched on cards on December 18, 1963 and required two people for 2.5 hours. Checking took two people another .83 hours. Correcting errors for the first trial required one person for .5 hour. The program was put on the computer for the first time on December 21, 1963. Four more trials were required on the computer before the results were satisfactory to send out to the farmer. Each time the problem needed correcting, approximately four hours were required to check the data cards and program for errors and repunch the necessary cards.

The first report was sent out to the farmer and District Agriculturist on January 7, 1964. A letter from the District Agriculturist was received on January 24, 1964, requesting some changes be made to the program.



These corrections were not made until April 9, 1964, and new data cards were punched to fit the format specified by Krieger<sup>39A</sup>. This required two people for 2.3 hours and checking for another .75 hours. Three trials were necessary on this program and the final letter was sent out on July 29, 1964. Computer time per trial on these programs could not be accurately measured, but was approximately two to four hours with the occasional trial taking eight hours of computer time. The matrix illustrated in Figure 4 and the results in Section 3.1.5 include the changes requested by the farmer.

### 3.2 CASE 2

#### 3.2.1 Resources.

The farmer in Case 2 had the following resources at his disposal:

- a) Land - arable            1,530 acres  
          - pasture        5,000 acres
- b) Livestock - cows        150 head  
              - feedlot capacity for 250 head of yearling  
                  steers at one time.
- c) Capital - land and buildings purchase        \$90,000.00  
              - livestock purchase                \$40,000.00
- d) Labor supply - operator plus two men year around.

#### 3.2.2 Farmer's Preferences.

This farmer was interested only in beef cattle and grain or hay crops.

#### 3.2.3 Data Inputs.

##### 3.2.3.1 Crop Yields and Prices.

These again consisted of a combination of the farmer's and District



Agriculturist's estimate and were largely the same as for Case 1. The only exception is that flax was not considered while rye on fallow was considered. The farmer estimated that rye on fallow would yield 15 bu/acre worth \$1.65/bu.

#### 3.2.3.2 Field Machinery.

The farmer's machinery is given in Table 18.

Table 18. Equipment Used By the Operator in Case 2.

Machine	Size or Model	Original Cost
Tractors	930 Case diesel	\$6,200.00
	560 Farmall diesel	\$5,500.00
Noble Blade	18 ft.	\$1,300.00
Press Drill	14 ft.	\$1,600.00
Cultivator (heavy duty with rodweeder attachment)	17 ft.	\$ 950.00
Rodweeder	12 ft.	\$ 700.00
Swather	15 ft.	\$1,550.00
Combines	151 I.H.C. (self-propelled)	\$6,500.00
	140 I.H.C.	\$3,800.00
Mower	7 ft.	\$ 600.00
Baler	I.H.C. 46	\$1,800.00
Rake	12 ft.	\$ 650.00

#### 3.2.3.3 Livestock Data.

Refer to the data for Case 1.







#### 3.2.3.4 Farming Practice.

Again summerfallow had to be on at least 1/3 of crop land. The operations on this farm were very similar to those in Case 1. Spring seeding on all land was done by cultivating, followed by a press drill. All crops were custom sprayed early in June and swathing was done in August. Combining was done late in August and early September. Tame hay was harvested in June and July with the sequence being mow, rake, bale, and stack. Summerfallow received the same three operations as Case 1. Grazing rates and livestock data were the same as that used in Case 1.

#### 3.2.3.5 Miscellaneous Information.

This information was the same as in Case 1, with the following added:

Baler twine cost	\$ .02/bale
Rye seed	\$2.00/acre.

#### 3.2.4 Results.

Gross Margin - \$55,600.00

#### Cropping Program

Purchase 1,050 acres arable land.

Durum wheat on fallow	- 860 acres
Red Spring wheat on stubble	- 860 acres
Fallow	- 860 acres



### Livestock Program

Buy 50 cows.

Fatten 180 of own calves and buy another 95 calves to fatten.

Feed:

- 310 acres cover crop.
- 5,382 bushels barley (purchased)
- 642 tons straw.
- 1,000 acres native range equivalent, used as winter pasture.

### Miscellaneous

- Rent pasture equivalent to 656 acres native range for winter pasture.
- Buy 585 tons of straw and pick up 300 tons from own land.

### 3.2.5 Remarks.

There are several questionable assumptions in this program. These assumptions are that

- a) Many activities similar to those in Case 1 had the same activity gross margin for this program even when the operation was slightly different.
- b) Fattening stock need not be separated according to sex.
- c) No limit was required on the amount of extra land which could be handled with the present machinery.
- d) No roughage restrictions were required for fattening animals.
- e) The buying and selling price of barley, oats, and other supplies need not be separated.
- f) Operating capital would not be limited.

Work on this program started on December 15, 1963 and the data in the matrix was ready to be put on cards on January 28, 1964. Punching cards



according to Wood's<sup>52</sup> program required 2 people for 2.0 hours and checking the cards took 2 people .33 hours. Three trials on the computer were required before the results were satisfactory. The initial results were mailed to the District Agriculturist and the farmer on February 20, 1964. On these trials, checking the program and correcting the necessary cards, required about 3.5 hours actual time. A return letter from the District Agriculturist received on March 5, 1964 provided the information for some changes. These changes were put in the program on April 26, 1964, and the entire problem punched to fit the format specified by Nichols and Krieger<sup>39A</sup>. Due to card punching errors, this again required three computer trials, so the results were not sent out until July 29, 1964. Checking time on each trial was about 2.5 to 3.0 hours.

### 3.3 CASE 3

#### 3.3.1 Resources.

The farmer in Case 3 had the following resources available:

- a) Land - arable      820 acres  
          - pasture     100 acres
- b) Livestock - cows    20 head  
              - feedlot capacity for 50 yearling steers at one time.
- c) Capital - operating                      \$15,000.00  
              - real estate purchase       \$30,000.00  
              - livestock purchase         \$60,000.00
- d) Labor supply - operator.

#### 3.3.2 Farmer's Preferences.

The operator here would consider raising durum wheat, Red Spring wheat, barley, flax, or oats. However, oats were left out of the program as they had exactly the same requirements as barley, and were less



profitable. As well, any beef cattle enterprise, any sheep enterprise, or raising hogs would be considered. An allowance would have to be made for the operator being unfamiliar with sheep. There were four rotations which he wished to consider and they were:

- a) Grain, fallow
- b) Grain, grain, fallow
- c) Grain, clover, grain, fallow
- d) Five years grass.

### 3.3.3 Data Inputs.

#### 3.3.3.1 Crop Yield.

In general, the data for crop yields were the same as for Case 1, with the exception of the following changes:

- a) The grain yield on land which had grown clover the previous year was regarded to be the same as after summerfallow.
- b) Clover hay yield was 1 ton of hay per acre worth \$22.00 per ton.
- c) Wheat yield on stubble land was reduced to 13. bushels per acre.

#### 3.3.3.2 Field Machinery.

This operator's machinery is given in Table 19.



Table 19. Equipment Used by Operator in Case 3.

Machine	Capacity or Model	Original Cost
Tractors	Oliver 1800 diesel	\$7,000.00
	Massey Ferguson 65 diesel	\$4,000.00
Cultivator and Rodweeder	7 acres/hr.	\$1,000.00 each
Discer and Press Drill	5 acres/hr.	\$1,500.00, discer
		\$2,000.00, drill
Sprayer	12 acres/hr.	\$ 400.00
Swather	8 acres/hr.	\$1,500.00
Combine	Oliver 33 (self-propelled)	
	6 acres/hr.	\$6,000.00
Cultivator or Rodweeder alone	8 acres/hr.	

### 3.3.3.3 Livestock.

#### 3.3.3.3.1 Beef Cattle:

The data for this program were the same as for Case 1 with the exception that the allowances for maximum cover crop consumption were changed to

- a) Cows - 5 acres
- b) Fattening calves - 1.5 acres
- c) Wintering calves - 3.0 acres
- d) Yearlings over summer - none
- e) Fattening long yearlings in winter - 2.0 acres

As well, the stocking rate for native pasture was reduced to 18 acres/cow and calf.



### 3.3.3.3.2 Sheep:

General husbandry data came from V. Gleddie<sup>22</sup>, Morrison<sup>38</sup>, and Sheep Production in Alberta<sup>43</sup>. The present feedlot was assumed to be adequate for sheep. The sheep were divided into the following:

#### a) Fatten lambs (60 lb. to 100 lb.):

##### - Feed requirements (Morrison<sup>38</sup>):

Average daily requirement = 2.2 lb. TDN, .22 lb. DP.

Feed required for a feeding period of 88 days  
194 lb. TDN, 19.4 lb. DP.

##### - Time:

.007 hr./day feeding, .2 hr. buying, and .2 hr. selling for each lamb.

##### - Selling price:

\$16.00/100 lb. buying and selling (Records from Love<sup>31</sup>).

##### - Costs:

2% loss	\$ .19
Vitamin and medicine	\$1.00
Shipping and yardage (two directions)	<u>\$1.31</u>
	\$2.50

##### - Activity gross margin:

\$16.00 - \$9.60 - \$2.50                      \$3.90

#### b) Raise lambs:

##### - Basis:

120% lamb crop (This is low but the farmer was not experienced).

##### - Time:

One man can look after 200 head of ewes. Six hours per ewe per year is the average labor requirement for sheep and includes time for fencing, haying, lambing,



worming etc. This was broken down to 3.7 hours required for winter feeding, lambing, worming, summer observation, and shipping lambs. 2.3 hours was for supplying the hay, fencing and duties which did not change with numbers.

- Feed (Morrison<sup>38</sup>):

Summer pasture  $\approx$  1/5 of beef cow requirement.

Winter 160 day feeding period  
360 lb. TDN, 33.2 lb. DP.

- Returns:

\$9.60 per lamb (60 lb. lamb sold) x 1.2 lambs/ewe  
\$11.40 per ewe for selling lambs. Selling wool  
\$6.00/fleece.

Total return	\$ 17.40
Costs - shearing	\$ .50
- veterinary fees	\$ .50
- selling and truck	\$ 1.00
- shipping fleece	<u>\$ 1.00</u>
	\$ 3.00
- Activity gross margin	\$ 14.40

3.3.3.3.3. Hogs:

- Basic unit: 10 sows raising 2 litters per year at 8 pigs per litter.

- Building requirement:

- Space*	8 ft. <sup>2</sup> /growing hog x 64 hogs	512
	65 ft. <sup>2</sup> /sow and litter x 2 sows	130
	35 ft. <sup>2</sup> /sow, no litter x 8 sows	280
	50 ft. <sup>2</sup> /boar x 1 boar	50
	Extras	<u>378</u>
		1,350 ft. <sup>2</sup>

\* Barre and Sammet<sup>4</sup> was the source for space requirements.



- Cost:

\$5,000.00 for insulated shell (local manufacturer).

\$2,500.00 for the equipment.

- Feed:

Feed efficiency 4.6 lb. feed/lb. gain.

(Source: Swine Production in Alberta<sup>49</sup>)

Ration: 720 lb. grain and 200 lb. supplement per hog sold at 200 lb. live weight. This includes the sow feed.

- Cost:

Feed grain 1.33¢/lb.

Supplement 5¢/lb.

Miscellaneous

\$ 22.00

\$ 3.00

\$ 25.00

- Return per hog:

\$25.00/100 lb. dressed weight x 150 lb. dressed weight

\$ 37.50

- Margin/hog to labor and fixed costs

\$ 12.50

#### 3.3.3.4 Farming Practices.

The rotation grain, clover, grain, fallow, deserves some explanation. The clover was seeded at a rate of 10 to 12 lbs. with the previous grain crop, and harvested for hay in June. This was then cultivated in July and summerfallowed the rest of the year. Fertilizer was applied only on land summerfallowed the previous year (50 lb. of 11-48-0 per acre), and on clover (65 lb. of 33.5-0-0 per acre). This farmer used tandem hitches in working his fields. All land was worked in late April with the cultivator and rodweeder in tandem. Then the crop was seeded in May using the discer, followed by the press drill in tandem. All crops were sprayed in June by the operator. Swathing was done in August and combining in



late August and early September. Any clover grown was swathed and baled in late June to enable the operator to start cultivation in mid July. Summerfallow land was worked once in May, once in early July (cultivator and rodweeder in tandem both times), and once in the first week of August (the rodweeder only). For the grass rotation, 1/5 of the acreage was reseeded each year. This necessitated breaking up 1/5 of the grass in July, summerfallowing until fall in one year, and reseeding the following spring.

### 3.3.3.5 Miscellaneous Data.

The following miscellaneous data were used in the analysis of Case 3:

- a) Labor cost     \$ 1.00 per hour.
- b) Fuel cost       \$ .18 per Imperial gallon, diesel.
- c) Spray cost      \$ 3.20 per gallon, 2,4D ester, 64 oz./gallon.
- d) Seed costs:
  - Wheat       \$ 2.50 per acre.
  - Barley      \$ 1.50 per acre.
  - Flax        \$ 4.00 per acre.
  - Durum      \$ 2.75 per acre.
- e) Fertilizer cost:
  - 11-48-0     \$100.00 per ton.
  - 33.5-0-0    \$ 75.00 per ton.
- f) Twine cost:
  - \$ .02 per bale, 40 bales/ton
- g) Land costs:
  - \$85.00 per arable acre, 5% interest charge.
  - \$60.00 per pasture acre, 5% interest charge.



[illegible]

Figure 6. Linear Programming Matrix Prepared for Case 3.



h) Livestock costs:

Feedlot space added	\$600.00 for 50 head.
Grass hay	\$ 20.00 per ton.
Beet pulp	\$ 45.00 per ton.
32% Supplement (beef)	\$ 86.00 per ton.

3.3.4 Results.

Gross Margin - \$25,000.00

Cropping Program

Buy 352 acres additional arable land.

Rotation - Grain, clover, grain, fallow - 1,172 acres.  
(Flax - 586 acres, clover - 293 acres, fallow - 293 acres).

Livestock

Buy 8 cows to raise 7 calves for fattening.

Buy 60 calves to bring the number in feedlot to 67 head.

Feed - 118 tons straw.  
1,897 bushels barley.

Buy 170 tons of straw for feed and bedding.

3.3.5 Remarks.

The questionable assumptions for this case study are the assumptions that

- a) All the beef cattle data could be transferred directly from Case 1. (This is questionable due to the change in the treatment of labor costs).
- b) An entry for barley to feed to hogs be made when the cost of barley had been part of the costs deducted from the gross margin to obtain the activity gross margin.
- c) An upper limit was not required for the amount of additional land which could be handled with the present equipment.
- d) No allowance was required for maintenance of the ewe herd or the ram.



There have been two improvements made to this program. The first is to leave the labor cost out of the partial budget and treat all labor, other than regular labor, as a uniform cost. Regular labor on the farm then became a fixed cost. The second improvement is to include a restriction for operating capital rather than ignore it.

The program was begun on February 1, 1964 and was ready for card punching on March 9, 1964. Punching the cards in the format specified by Wood<sup>53</sup> required two people for 2.0 hours and checking took two people .33 hours. The program required three trials before it was satisfactory. This was mainly due to errors in the program preparation. The initial results were sent to the farmer on April 2, 1964, and no changes were requested. Rechecking for errors required about 3.5 hours in each new trial.

### 3.4 CASE 4

#### 3.4.1 Resources.

The farmer in this case study had the following resources available:

- a) Land - Arable - 484 acres
  - Pasture - 120 acres (40 acres could be broken up)
- b) Livestock - Feedlot space - 75 yearling steers at once.
  - Feedlot expansion limited to capacity for another 75 head of yearling steers.
  - Barn space which could be renovated to care for 20 sows and litters to market.
- c) Capital available - Operating - \$14,000.00
  - Livestock purchase - \$13,000.00
  - Real Estate purchase - \$30,000.00
- d) Labor supply - Operator.



### 3.4.2 Farmer's Preferences.

The operator would consider raising grain, hay, hogs, sheep, and any beef cattle enterprise. He did not want any summerfallow in the grain rotation and would consider having up to one-half of his arable land in forage crops. Grain crops could not be seeded on the same land in two consecutive years. He wished to consider the following rotations:

- a) Continuous grain
- b) Four years grain, one year clover
- c) Four years grain, four years grass (two years hay, two years pasture)

### 3.4.3 Data Inputs.

#### 3.4.3.1 Crop Yields and Prices.

Table 20 summarizes the farmer's estimates.

Table 20. Data on Crop Yields and Prices Used in Case 4.

Crop	Yield		Farm Price
	On Land Previously Seeded to Hay	On Land Previously Seeded to Grain	
Wheat	25 bus/acre	23 bus/acre	\$ 1.45/bu
Barley	35 bus/acre	32 bus/acre	\$ .90/bu
Oats	50 bus/acre	46 bus/acre	\$ .55/bu
Hay	1 ton/acre	1 ton/acre	\$18.00/ton

#### 3.4.3.2 Field Machinery.

This operator's equipment is given in Table 21.



Table 21. Equipment Used by Operator in Case 4.

Machine	Size or Model	Original Cost
Tractor	Cockshutt P.D. 40 Diesel	\$4,000.00
Discer	12 ft	\$1,500.00
Cultivator (rodweeder attachment)	12 ft	\$ 800.00
Press drill	12 ft	\$2,000.00
Plow ( $\frac{1}{2}$ share)	4 - 14" bottoms	\$ 300.00
Harrows	25 ft	\$ 500.00
Rodweeder	12 ft	\$ 800.00
Fertilizer Spreader ( $\frac{1}{2}$ share)	12 ft	\$ 250.00
Tractor and Front End Loader (2/3 share)	Farmall M	Tractor - \$3,000.00 Loader - \$ 800.00
Sprayer ( $\frac{1}{2}$ share)	46 ft	\$ 450.00
Swather	12 ft	\$1,200.00
Combine (self-propelled)	25 acres/day	\$5,500.00
Mower	6 ft	\$ 460.00
Rake	7 ft	\$ 300.00
Hammermill		

### 3.4.3.3 Livestock.

#### 3.4.3.3.1 Beef cattle:

The data used for Case 4 was the same as for Case 1 with the exception that the price of calves sold was reduced to \$ .24/lb. As well, the partial budget did not include a labor charge. The activity gross margin per cow was \$92.20.



3.4.3.3.2 Sheep:

The data used for Case 4 was identical to that used for Case 3.

3.4.3.3.3 Hogs:

The farmer gave relatively complete information on this enterprise as he was familiar with raising hogs. In order to obtain a figure for a new building cost, a basic unit of 20 sows farrowing continuously was selected. The building costs are as follows:

a) New building (revised from Case 3):

- Space (Barre & Sammet<sup>4</sup>)

Boar	-	50 ft <sup>2</sup>
4 sows and litters (65 ft <sup>2</sup> )	-	260 ft <sup>2</sup>
16 sows (30 ft <sup>2</sup> )	-	480 ft <sup>2</sup>
43 hogs under 100 lb. (4 ft <sup>2</sup> )	-	172 ft <sup>2</sup>
85 hogs over 100 lb. (8 ft <sup>2</sup> )	-	680 ft <sup>2</sup>
Extra	-	<u>158 ft<sup>2</sup></u>
		1,800 ft <sup>2</sup>

- Value of building	\$3,600.00
- Value of equipment	<u>\$1,400.00</u>
	\$5,000.00
- Value per sow in building	\$ 250.00

b) Renovate old barn:

\$25.00 per sow and litter fattened.  
Capacity = 20 litters.



The following activities were considered:

a) Raise and fatten pigs.

- Unit = 1 sow, sell 16 pigs/year.
- This operation would be the new specific pathogen free pattern. Therefore the sow value increased from \$64.00 to \$80.00. Space requirement for the sow unit includes space for fattening animals.
- Basis for feed: 4 lb. feed/lb. gain.

Feed 800 lb./200 lb. hog.

720 lb. grain, 80 lb. 35% supplement.

Supplement cost per unit	\$ 71.68
--------------------------	----------

Miscellaneous cost	<u>\$ 39.50</u>
--------------------	-----------------

\$111.18

- |  |          |
|--|----------|
| - Gross return per unit<br>(150 lbs. x \$24.00/100 lbs. x 16 hogs) | \$576.00 |
|--|----------|

- |                                  |          |
|----------------------------------|----------|
| - Activity gross margin per unit | \$465.00 |
|----------------------------------|----------|

b) Raise and sell weanlings.

- Unit = 1 sow, sell 16 weanlings/year.
- This operation would also be the specific pathogen free pattern and would be the same as the above operation except for the fattening.
- Feed for sow - 2,200 lb. grain, 300 lb. 35% supplement.

Supplement cost	\$ 16.80
-----------------	----------

Miscellaneous cost	<u>\$ 19.25</u>
--------------------	-----------------

\$ 36.05

- |   |          |
|---|----------|
| - Gross returns per unit<br>(16 x \$12.00/weanling) | \$192.00 |
|---|----------|

- |                                  |          |
|----------------------------------|----------|
| - Activity gross margin per unit | \$156.00 |
|----------------------------------|----------|



c) Fatten weanlings.

- Unit = 27 hogs.
- This operation would necessitate purchase of weanlings from other farmers and thus a lower price for weanlings would be charged.

- Feed: 351 bushels of barley.  
1,755 lbs. 35% supplement.

- Costs:

Hogs (27 hogs x \$10.00/hog)	\$270.00
Supplement	\$ 98.28
Miscellaneous	<u>\$ 30.00</u>
	\$398.28

- Gross returns per unit  
(\$35.00/hog x 27 hogs) \$945.00
- Activity gross margin per unit \$547.00

3.4.3.4 Farming Practice.

This farmer did not summerfallow so there was no operation to look after fallow. All grain land was worked with a discer early in May or late April. Then after waiting for wild oats to sprout, he seeded with the press drill following the cultivator or rodweeder. This was then harrowed to smooth the field. All crops were sprayed in June and swathed in August. Combining was done in September and all grain land was fall cultivated. Any land in clover was seeded with the previous grain crop and left only one year when it was taken off as hay. Grassland was seeded in the spring and left for four years; the first two years for hay and the second two years for pasture. Fertilizer application was as follows:



- a) Grain land - fall - 100 lbs./acre, 33.5-0-0  
- spring - 40 lbs./acre, 11-48-0
- b) Grass land - 100 lbs./acre, 33.5-0-0
- c) Clover land - 80 lbs./acre, 16-20-0

Clover land was cultivated after the hay crop and summerfallowed for the remainder of the year with the sequence of operations being plow, disc, and rodweed. Grass land was plowed after mid-summer, and disced twice before freeze-up. Then in the spring it received an extra cultivation.

#### 3.4.3.5 Miscellaneous Data.

The following miscellaneous data were used in the analysis of Case 4.

- a) Labor cost - \$1.00 per hour
- b) Spray cost -2,4D Ester \$3.20/gallon  
(64 oz. of acid/gallon)  
- MCP Amine \$5.00/gallon  
(80 oz. of acid/gallon)
- c) Fertilizer cost - 11-48-0 \$100.00/ton  
- 33.5-0-0 \$ 65.00/ton  
- 16-20-0 \$ 74.00/ton
- d) Seed costs - Clover \$3.00/acre  
- Grass \$4.50/acre  
- Wheat \$2.25/acre  
- Barley \$1.60/acre  
- Oats \$1.20/acre
- e) Cost of additional arable land - \$110.00/acre.  
5% interest charge
- f) Fuel cost (diesel) - \$ .186/Imperial gallon.
- g) Livestock data (miscellaneous):
  - 32% beef cattle supplement - \$ 90.00/ton
  - cost of beef feedlot increase  
(to add 75 head capacity) - \$750.00
  - cost of barn renovation for 20 sows \$500.00  
and litters -
  - straw is piled with front end loader.







- Time for swine:

Three hours per day to look after 20 sows with the litters. Assume half the time was spent fattening the litter from 35 lb. weanling and half the time spent to keep the sow and raise a 35 lb. weanling.

3.4.4 Results.

Gross Margin - \$29,977.00

Cropping Program

- Buy 129 acres and break 40 acres
- Crops - barley    327 acres
- wheat     327 acres.

Livestock

- Sell weanling pigs from 162 sows.
- Build new barn for 122 sows.
- Renovate old barn for 40 sows.

Miscellaneous

- Hire one man for full time employment and an extra man for the summer.
- Pick up 81 tons of straw for bedding.

3.4.5 Remarks.

Since the data for beef cattle and sheep activities came directly from previous case studies, the questionable assumptions which applied previously also apply in this case study. In addition, the following assumptions are



debatable:

- a) That direct transfer of livestock data from previous case studies was satisfactory.
- b) That the depreciation cost of the swine barn and feedlot was not linear, so it should be left out of the activity cost calculations.
- c) That an upper limit was not required on the amount of land which could be farmed with the present machinery.

The improvements in this program were principally in the cropping activities and the swine activities. Rounding of costs was not done until the activity gross margin had been calculated. The swine activities included an allowance for the depreciation of the sow.

Program formulation began June 2, 1964 and was completed on June 12, 1964. Card punching required one person for 4.83 hours and card checking required one person for .83 hours. Five trials on the computer were required before the results were satisfactory. Correction time for each trial required approximately three hours. The results were sent out on October 7, 1964, and there were no requests for changes.

### 3.5 CASE 5

#### 3.5.1 Resources.

The farmer in Case 5 had the following resources at his disposal:

- a) Arable land - 390 acres.
- b) Feedlot capacity - 300 yearling steers at one time.
- c) Capital - Operating - \$30,000.00  
- Real Estate Purchase - \$10,000.00  
- Livestock Purchase - \$60,000.00
- d) Permanent labor - Operator.



### 3.5.2 Farmer's Preferences.

This farmer was concerned primarily with the type of animal he should fatten. He felt the returns from grain production were not satisfactory for the small amount of land he owned and he did not want to buy any more.

### 3.5.3 Data Inputs.

#### 3.5.3.1 Crop Yields and Prices.

The farmer estimated his yields to be:

- a) Oat hay - 2 ton /acre worth \$15.00/ton
- b) Legume hay - 1 ton/acre worth \$30.00/ton
- c) Oat silage - 6 ton/acre
- d) Legume silage - 3 ton/acre

#### 3.5.3.2 Field Machinery.

The machinery used by the operator in Case 5 is given in Table 22.

Table 22. Basic Field Machinery Data for Case 5.

Machine	Size or Model	Original Cost
Tractors	Caterpillar 2-3 plow	
	4 plow	\$4,400.00
	5 plow	\$5,400.00
Seed drill	12 ft.	\$2,000.00
Harrows	40 ft.	\$ 500.00
Cultivator	12 ft.	\$1,000.00
Disk	12 ft.	\$1,100.00
Forage Harvester	7.5 ft.	\$2,400.00
Roller		
Silage unloading wagon		
Manure and silage wagon		



### 3.5.3.3 Livestock.

#### 3.5.3.3.1 Beef cattle:

For Case 5 the beef cattle were separated with regard to sex. All steers being fattened were the only stock which received implants. Table 23 contains the summary of the data used in Case 5.

#### 3.5.3.3.2 Hogs:

The data for feeder hogs in Case 4 was transferred directly with the assumption that this activity applied to summer only.

### 3.5.3.4 Farming Practice.

All land in oats was cultivated twice after the crop was harvested. Hay land was broken up after the hay crop was harvested using the cultivator twice and the tandem disc four times. Seeding was done in the spring by cultivating the land, followed by the drill and packers. The oat crop was then sprayed in June and harvested in July or August. Fertilizer was applied in the spring at the rate of 110 lb. per acre of 27-14-0 on oats and 100 lb. per acre of 33.5-0-0 on any grass land. Hay harvesting required the use of custom equipment.

### 3.5.3.5 Miscellaneous Information.

The following information on miscellaneous data was used.

- a) Straw purchased - \$10.00/ton
- b) Labor cost - \$1.00/hour
- c) Barley cost - unrolled     \$ .85/bushel  
                  - rolled        \$ .85/bushel and \$2.00/ton
- d) Seed cost - oats         \$3.00/acre  
                  - grass      \$5.20/acre



Table 23. Summary of Budgets for Beef Cattle in Case 5.

Class	Buying Cost	Variable Cost		Gross Returns	Activity Gross Margin
		Except for	Feed and Labor		
Steer calves fattened in winter (180 days)	\$112.50 (450 lb x \$ .25/lb)	\$13.94	\$204.30 (900 lb x \$ .227/lb)	\$67.86	
Heifer calves fattened in winter (174 days)	\$ 88.00 (400 lb x \$ .22/lb)	\$11.36	\$160.00 (800 lb x \$ .20/lb)	\$60.64	
Yearling steers fattened in summer (140 days)	\$132.00 (600 lb x \$ .22/lb)	\$14.39	\$217.08 (950 lb x \$ .2285/lb)	\$70.69	
Yearling heifers fattened in summer (134 days)	\$110.00 (550 lb x \$ .20/lb)	\$11.86	\$178.50 (850 lb x \$ .21/lb)	\$56.74	
Long yearling steers fattened in winter (114 days)	\$168.00 (800 lb x \$ .22/lb)	\$14.21	\$247.50 (1,100 lb x \$ .225/lb)	\$65.29	
Steer calves; grow 180 days (winter) and fatten 140 days (summer)	\$112.50 (450 lb x \$ .25/lb)	\$11.79	\$213.75 (950 lb x \$ .225/lb)	\$89.56	
Heifer calves, grow 180 days, fatten 130 days	\$ 88.00 (400 lb x \$ .22/lb)	\$11.86	\$187.00 (850 lb x \$ .22/lb)	\$87.14	
Long yearling steers, grow 180 days, fatten 82 days	\$168.00 (800 lb x \$ .21/lb)	\$15.46	\$228.75 (1,150 lb x \$ .225/lb)	\$75.29	







- e) Fertilizer cost - 27-14-0    \$88.00/ton
- 33.5-0-0    \$80.50/ton

f) Miscellaneous livestock data:

- Time on cattle:

- Base of estimate (300 head growing in winter,  
fattening in summer)

- Growing - 1.5 hours/day feeding
- .5 hour/week grinding.

- Fattening - put on self feed and pasture
- 10 hours/week for grinding.

- Miscellaneous time:

- Implant - 2 men for three days.

- Vaccinate - 1 man for 1 day.

- Corral cleaning - 2 men for one month of year.

- Bedding - 40 lb/animal/week in winter.
- 20 lb/animal/week in summer.
- Costs - 32% beef supplement - \$90.00/ton.
- yardage and commission rates as for Case 1.

#### 3.5.4 Results.

Gross Margin - \$32,711.00

#### Cropping Program

- 279 acres legume silage
- 56 acres oat silage
- 56 acres oat pasture.

#### Livestock Program

- 39 steer calves fattened in winter.
- 336 heifer calves grown during winter and fattened in summer.
- 197 yearling steers fattened during summer.



Feed:

- Winter: Oat hay - 467 ton (purchased)  
Legume silage - 121 ton
- Summer: Oat pasture - 56 acres  
Legume silage - 715 ton

Miscellaneous

- Some casual labor is required for April, July and August.
- Buy 26 tons straw for bedding.

3.5.5 Remarks.

Cursory examination of the results of this program may leave some questions about the program, due to the high roughage ration. The cause of the high roughage ration was the assumption that there were really no restrictions on the amount of roughage fattening animals could consume and still maintain the suggested gains. However, the results of the program satisfy all the conditions as the problem was formulated.

This program was improved substantially over the previous case studies. Two things were done, as follows:

- a) Beef animals were separated with regard to sex.
- b) Labor costs were left out of the activity cost calculations.

Program preparation began on June 13, 1964 and was completed on July 13, 1964. Data preparation for the first trial required one person for 3.75 hours. Four trials with correction time of approximately three hours on each trial were required to get the results. The results were sent out on July 30, 1964, and no re-runs were requested.



### 3.6 CASE 6

### 3.6.1 Resources.

The farmer in Case 6 had the following resources:

- a) Land - arable - 280 acres owned  
             - 580 acres rented  
             - pasture - 50 acres
- b) Flax limit - 290 acres.
- c) Feedlot capacity - 80 yearling steers at one time.
- d) Swine capacity - 20 sows and litters.
- e) Capital - Operating                     - \$ 9,000.00  
             - Livestock purchase       - \$22,000.00  
             - Real Estate purchase - \$20,000.00
- f) Labor supply - Operator

### 3.6.2 Preferences.

This farmer would consider any of the usual crop and livestock enterprises which had been tried in the area and found feasible.

### 3.6.3 Data Inputs.

#### 3.6.3.1 Crop Yields and Prices.

A brief summary of the basic data used for Case 6 is given in Table 24.



Table 24. Yields and Prices Used in Case 6.

Crop	Yield		Farm Price	
	On Fallow	On Stubble	Sold	Purchased
Wheat	21 bus/acre	14 bus/acre	\$ 1.45/bu	
Barley	30 bus/acre	18 bus/acre	\$ .82/bu	\$ .83/bu
Oats	40 bus/acre	25 bus/acre	\$ .55/bu	\$ .555/bu
Flax	14 bus/acre		\$ 2.80/bu	
Durum Wheat	21 bus/acre		\$ 1.75/bu	
Hay		.5 ton/acre/year	\$20.00/ton	\$20.00/ton
Straw				\$ 9.00/ton

### 3.6.3.2 Field Machinery.

The tractor size and model was not obtained from the farmer so costs per acre listed in Table 25 were taken from previous programs.

Table 25. Equipment Costs Used for Case 6.

Machine	Cost Per Acre	Time Per Acre
Discer and Packers	\$ .35	.25 hr.
Discer	\$ .27	.20 hr.
Noble Blade	\$ .50	.33 hr.
Harrow	\$ .11	.12 hr.
Spray	\$ .05	.05 hr.
Swather (grain)	\$ .25	.15 hr.
(hay)	\$ .60	.45 hr.
Combine	\$ .75	.24 hr.
Cultivate	\$ .30	.20 hr.



### 3.6.3.3 Livestock.

Activities for sheep were transferred from Case 3 and activities for hogs were transferred from Case 4. A summary of the budgets for fattening stock is given in Table 26 while the following calculations were used to determine activity gross margins for the operators cow-calf alternatives.

#### a) Sell feeder calves:

- Gross returns - .46 steers/cow (475 lb/steer x \$ .25/lb.)	\$ 54.625
- .46 heifers/cow (440 lb/heifer x \$ .22/lb.)	<u>\$ 44.528</u>
	\$ 99.153
- Costs:	
- Cow - losses, depreciation, minerals, veterinary and medicine	\$ 12.00
- .92 calf - trucking, creep, commission	<u>\$ 3.81</u>
Total	\$ 15.81
- Activity gross margin	\$ 83.34
- Cow value	\$160.00

#### b) Sell registered stock:

This activity was identical to a) with the exception of the following alterations:

- Sell 1/3 of crop for \$72.00/calf more than a).	
- Keep 1/3 of herd for extra three months	\$ 13.50/head kept
- Register 1/3 of herd	\$ 3.00/head registered
- Extra return/calf in herd	\$ 24.00
- Extra cost/calf in herd	<u>\$ 5.50</u>
- Additional return to feed etc./calf	\$ 18.50
- Additional return to feed etc./cow (.92 x 18.50)	\$ 17.02



Table 26. Budgets for Fattening Stock in Case 6.

Stock and Feeding Period	Variable Costs		Activity Gross Margin
	Buying Costs	Feed and Labor	
Steer calves fattened over winter	\$117.00 (450 lb x \$ .26/lb)	\$14.83	\$213.30 (900 lb x \$ .237/lb) \$81.47
Heifer calves fattened over winter	\$ 92.00 (400 lb x \$ .23/lb)	\$11.96	\$168.00 (800 lb x \$ .21/lb) \$64.04
Steer calves grown through winter and fattened in summer	\$117.00 (450 lb x \$ .26/lb)	\$13.99	\$216.00 (900 lb x \$ .24/lb) \$85.01
Heifer calves grown and summer fattened	\$ 92.00 (400 lb x \$ .23/lb)	\$12.34	\$178.50 (850 lb x \$ .21/lb) \$73.16
Fatten yearling steers over summer	\$138.00 (600 lb x \$ .23/lb)	\$15.51	\$232.75 (950 lb x \$ .245/lb) \$79.24
Fatten yearling heifers over summer	\$115.50 (550 lb x \$ .21/lb)	\$12.46	\$187.00 (850 lb x \$ .22/lb) \$59.04
Fatten long yearling steers over winter	\$176.00 (800 lb x \$ .22/lb)	\$16.06	\$264.00 (1,100 lb x \$ .24/lb) \$71.94
Grow long yearling steer through winter and fatten in summer	\$176.00 (800 lb x \$ .22/lb)	\$14.62	\$276.00 (1,150 lb x \$ .24/lb) \$85.38



- Activity gross margin ( $\$83.34 + \$17.02$ ) =  $\$100.36$

#### 3.6.3.4 Farming Practices.

All crops were seeded early in May with the discer and packers (one operation), followed by the harrows (separate operation). In June, all crops were sprayed. Swathing was done in August and combining in September. Summerfallow land received three workings: Discer in May, Cultivator in July, and Noble Blade in late July. Hay was swathed and custom baled. Fertilizer application on hay was 100 lb/acre of 33.5-0-0; on land previously fallowed was 40 lb/acre of 11-48-0; and on land previously cropped was none.

#### 3.6.3.5 Miscellaneous Data.

Data on cover crop was obtained from Case 1 and data for baling straw came from Case 3. In addition, the following costs were used:

- a) Labor cost -  $\$1.00/\text{hour}$
- b) Fertilizer cost - 11-48-0 =  $\$100.00/\text{ton}$   
- 33.5-0-0 =  $\$75.00/\text{ton}$
- c) Land cost-Capital - Buy the rented land =  $\$70.00/\text{acre arable}$   
- Buy other land =  $\$80.00/\text{acre arable}$   
-Operating - 5% interest charge.
- d) Rental terms: 1/3 of grain crop or hay crop of 1/2 the value of tame pasture.
- e) Livestock costs - Yardage =  $\$1.26/\text{head sold}$   
- Commission - selling =  $\$1.50/\text{head over } 500 \text{ lb.}$   
=  $\$.75/\text{head under } 500 \text{ lb.}$   
=  $\$.15/\text{hundred lbs.}$   
  
- Freight =  $\$.45/100 \text{ lbs. to Calgary.}$
- f) Pasture capacity of tame pasture - 10 acres/cow and calf.
- g) Spray data:
  - Cost -  $\$3.52/\text{gallon for } 2,4\text{D Ester (64 oz. acid/gallon)}$
  - rate of application =  $5.5 \text{ oz/acre.}$







#### 3.6.4 Results.

Gross Margin - \$14,962.00

#### Cropping Program

- Buy 262 acres of rented land and 19 acres other land to bring owned arable acreage to 563 acres.

- Crops:

- Owned land - 269 acres flax
    - 25 acres barley
    - 269 acres fallow

- Rented land - 205 acres - 2 years grass, 2 years grain
    - 111 acres - 4 years grass, 1 year oat nurse crop cut for hay.
    - Grass - 190 acres hay
      - 79 acres tame pasture
    - Grain
      - 8 acres oats
      - 36 acres wheat
      - 58 acres barley

#### Livestock Program

- Number and class:

- 62 yearling steers fattened over winter.
  - 80 long yearling steers fattened over summer.

- Feed:

- Winter
    - 4 ton oats
    - 35 ton barley
    - 34 ton straw
    - 27 ton hay
  - Summer
    - 70 ton hay
    - 79 acres tame pasture
    - 50 acres native pasture

#### Miscellaneous

- Pick up 70 ton of straw.



### 3.6.5 Remarks.

One assumption may be questioned in this study. This is the assumption that there does not need to be a roughage intake limit on fattening animals.

The main change resulting in a better program here was in the allowance for herd replacements in the cow herd. This was done by placing a depreciation cost on the cow. As well, a limit was placed on the straw intake of the animals and the farm price for crops purchased was raised above the farm price for the same crops sold.

The program was started on July 14, 1964 and was ready for punching on July 22, 1964. Card punching and checking for the initial run required 3.33 hours for one person. This program required two trials before it was completed and it was ready for analysis on September 8, 1964. The first check only took .5 hour, the second required 3.5 hours for a complete check of coding and punching. Analysis required another four hours, but this was not done until September 30, 1964 when the results were sent to the District Agriculturist in the area. A rerun was not requested.

## 3.7 CASE 7

### 3.7.1 Resources.

The following resources were available:

- a) Land - arable - 955 acres  
- pasture - 240 acres
- b) Feedlot capacity - 600 yearling steers at one time
- c) Labor supply - 3 men year around.



d) Capital - Operating - \$ 50,000.00  
- Livestock Purchase - \$140,000.00  
- Real Estate Purchase - \$150,000.00

### 3.7.2 Farmer's Preferences.

This operation was a partnership and the management was interested only in beef production of any type, or crop production limited to wheat, oats, barley or forage crops. They did not feel summerfallow was necessary.

### 3.7.3 Data Inputs.

#### 3.7.3.1 Crop Yields and Prices.

Most of the data in Table 27 came from past records supplied by the operators in Case 7.

Table 27. Crop Data for Case 7.

Crop	Yield	Farm Price	
		Sold	Purchased
Wheat	35 bus/acre	\$ 1.40/bu	
Oats	63 bus/acre	\$ .55/bu	\$ .555/bu
Barley	52 bus/acre	\$ .82/bu	\$ .83/bu
Hay	1.4 ton/acre	\$24.50/ton	\$25.00/ton
Silage - legume	4.2 ton/acre		
- oat	5.8 ton/acre		

#### 3.7.3.2 Field Machinery.

Table 28 gives the machinery data used in Case 7.



Table 28. Basic Field Machinery Data for Case 7.

Machine	Size or Model	Original Cost
Tractors	I.H.C. 660 Diesel	\$5,800.00
	I.H.C. 450 Diesel	\$4,300.00
Cultivator	15 ft.	\$1,200.00
Discer	16 ft.	\$2,000.00
Drill	20 ft.	\$3,500.00
Harrow	50 ft.	\$ 600.00
Swather	} - Refer to Case 6.	
Combine		
Forage Harvester - Refer to Case 5.		

#### 3.7.3.3 Livestock.

The beef cattle data were obtained directly from the activities in Case 6.

#### 3.7.3.4 Farming Practices.

Land in grain crops was all fall cultivated with the heavy duty cultivator. All crop seeded in the spring was worked with the discer very early, the discer or cultivator two weeks later, and the discer in late May, followed immediately by the drills. All land was then harrowed, followed by spraying in late June. Swathing was done in late August, and combining in September. Barley was not seeded on the same land for more than two consecutive years. At least 30% of the land had to be in forage and at least 10% would be necessarily in an oat nurse crop. Fertilizer rates were:

- a) 100 lb/acre - 33.5-0-0 in the fall and 50 lb/acre of 11-48-0 at seeding time for all grain crops.







- b) 200 lb/acre - 16-20-0 in early spring or late fall on forage crops.

#### 3.7.3.5 Miscellaneous Data.

The following data were also supplied:

- a) Labor cost - \$1.00/hour
- b) Livestock cost:
  - New feedlot space - \$34.60/yearling space added.
  - 32% supplement - \$100.00/ton.
  - Straw - \$2.20/ton to bale own straw
  - \$9.00/ton to deliver to the farm.
- c) Additional land - \$110/acre arable.
- d) Fuel cost - \$ .18/Imperial gallon for diesel.
- e) Fertilizer cost - 33.5-0-0    \$ 65.00/ton  
                              - 11-48-0        \$100.00/ton  
                              - rent spreader - \$ 2.00/ton
- f) Baler twine - \$ .02/bale.

#### 3.7.4 Results.

Gross Margin - \$100,900.00

##### Cropping

- Buy 1,250 arable acres
- 441 acres wheat
- 882 acres barley
- 221 acres oats (nurse crop for grass)
- 409 acres legume hay
- 170 acres legume silage
- 82 acres tame pasture

##### Livestock

- Fatten - 369 steer calves in winter
- 696 yearling steers in summer.



Feed:

- Winter - 62 ton barley
  - 212 ton oats
  - 714 ton legume silage
  - 155 ton straw
- Summer - 564 ton barley
  - 24 ton oats
  - 240 acres rough pasture
  - 82 acres tame pasture

Miscellaneous

Sell - 476 ton barley  
- 573 ton legume hay

Buy - space for 96 more yearling steers in feedlot.

Pick up 479 tons straw.

3.7.5 Remarks.

The main criticisms of the program are:

- a) not enough data were obtained from the operator on the machinery and
- b) a restriction was not placed on the amount of additional land which could be farmed.

In this program, a restriction has been added to improve the ration formulation but a specific winter roughage limit was not made for cows.

This program was begun on July 22, 1964 and was ready for the card preparation on July 27, 1964. Card preparation took four hours (punching and checking). Three trials on the 1620 were completed, but punching errors made the results unsatisfactory. Each time checking required two hours. The last trial was completed on October 6, 1964, and the 1620 computer was removed immediately after this. The next 2½ months were spent in waiting for the new 7040 computer and it was not until February 9, 1965 that this problem was punched in the format specified by Clasen<sup>13</sup>.



This time, data preparation only required 2.75 hours. However, four trials were required on the new computer before results were satisfactory. Two trials were the result of adding additional restrictions to bring the beef cattle ration into a more reasonable area. The computer results were tabulated and included in a letter with the data used. This preparation took about three hours and the results went out to the District Agriculturist on March 10, 1965. This was followed by a meeting with one partner on April 5, 1965 to discuss the results. No changes were requested and the farm operator felt the results were very valuable.

### 3.8 CASE 8

#### 3.8.1 Resources.

This farmer had rather limited resources compared to the others in the study. His resources are as follows:

- a) Arable land - 250 acres.
- b) Labor - Operator year around.
- c) Capital - Operating - \$ 4,000.00  
- Livestock - \$ 1,000.00  
- Real Estate - \$20,000.00
- d) Barn space - renovations possible to hold 10 sows and litters year around.

#### 3.8.2 Preferences.

This farmer was interested in almost any feasible enterprise in the area. His prime interest was in grain or forage seed production and beef.

#### 3.8.3 Data Inputs.

##### 3.8.3.1 Crop Yields and Prices.

The grain yields and prices were based on the farmer's estimates



while the forage crop yields came from C.R. Elliot<sup>18</sup> at the Research Station at Beaverlodge. The prices for the forage seeds were based on the conservative prices from the trade. The yields and prices used in the program are given in Table 29.

Table 29. Crop Yields and Prices Used in Case 8.

Crop	Yield			Farm Price	
	Grain On Fallow	Grain On Stubble	Grass	Sold	Purchased
Wheat	31 bus/acre	27.4 bus/acre		\$ 1.40/bu	
Barley	85 bus/acre	40 bus/acre		\$ .80/bu	
Flax	14 bus/acre			\$ 2.80/bu	
Rapeseed			1000 lb per acre	\$ .022/lb	
Alsike clover seed			200 lb per acre	\$ .13/lb	
LaSalle clover seed			120 lb per acre	\$ .24/lb	
Fescue seed			200 lb per acre	\$ .20/lb	
Fescue aftermath			1.2 ton per acre	\$10.00/ton	
Grass hay			1.5 ton per acre	\$10.50/ton	\$10.00/ton

### 3.8.3.2 Field Machinery.

The field machinery data for this case study are found in Table 30.



Table 30. Basic Field Machinery Data for Case 8.

Machine	Size or Model	Original Cost
Tractor	Cockshutt 570 Diesel	\$5,000.00
Cultivator	12 ft.	\$ 800.00
Discer and Seeder Attachment	12 ft.	\$2,100.00
Harrows	24 ft.	\$ 600.00
Sprayer	32 ft.	\$ 200.00
Swather	12 ft.	\$1,200.00
Double Disc Drill	10 ft.	\$1,800.00
Plow	3.5 ft. (3-14")	\$ 400.00
Combine	Case 120	\$6,000.00

#### 3.8.3.3 Livestock.

This farmer was only interested in beef and swine. The data used areas follows:

##### a) Beef:

The feeding period and nutrient requirements are the same as Case 7. Labor requirements and the cost of feeding were adjusted to the area. The essential economic data are shown in Table 31.

##### b) Hogs:

The data from Case 4 for the majority of costs, time, and feeding requirements were used in this program. The only changes are that weanlings are sold at \$10.00/weanling, the freight per finished hog sold is \$1.50 per hog more than in Case 4, and the returns per finished hog are \$36.00.



Table 31. Returns and Cost Data for the Beef Cattle Activities Considered in Case 8.

Class	Buying Cost	Variable Cost		Gross Returns	Activity Gross Margin
		Except Feed and Labor			
Cows - sell calves 95% calf crop		\$10.00/cow \$ 5.41/calf	.475 steer (475 lb x \$ .22/lb) .475 heifer (440 lb x \$ .20/lb)	\$96.25	\$76.30
Steer calves fattened in winter	\$ 99.00 (450 lb x \$ .22/lb)	\$18.11	\$180.00 (900 lb x \$ .20/lb)		\$62.29
Heifer calves fattened in winter	\$ 80.00 (400 lb x \$ .20/lb)	\$15.36	\$148.00 (800 lb x \$ .185/lb)		\$52.64
Yearling heifers fattened over summer	\$104.50 (550 lb x \$ .19/lb)	\$16.25	\$153.00 (850 lb x \$ .18/lb)		\$32.25
Yearling steer fattened over summer	\$126.00 (600 lb x \$ .21/lb)	\$19.46	\$190.00 (950 lb x \$ .20/lb)		\$44.54
Buy heifer calves, grow through winter and fatten in spring	\$ 80.00 (400 lb x \$ .20/lb)	\$15.36	\$153.00 (850 lb x \$ .18/lb)		\$57.64
Buy steer calves, grow through winter, fatten in summer	\$ 99.00 (450 lb x \$ .22/lb)	\$18.05	\$190.00 (950 lb x \$ .20/lb)		\$72.95



#### 3.8.3.4 Farming Practice.

All land to be seeded to grain was cultivated and harrowed early in May. Then the crop was seeded using the drill, followed by the harrows in mid May. The crop was sprayed in June. Combining was done in September with only the wheat being swathed before combining. This land was then worked with the discer after combining. Fallow land was worked in May with the cultivator, in June with the discer, in July with the cultivator, and in September with the discer. The land was harrowed after each cultivation. Clover crops harvested for seed were combined as a standing crop in late September and early October. Any of this land to be reseeded the following year was then plowed. Creeping Red Fescue was combined as a standing crop in July and subsequent re-growth could be harvested for hay in September. Hay was harvested in June and July by swathing and hiring a custom operator to bale it. His fertilizer practice was as follows:

Crops on fallow	- 55 lb/acre - 11-48-0
Stubble crops	- 75 lb/acre - 27-14-0 broadcast
	- 60 lb/acre - 16-20-0 with seed
Grass	-100 lb/acre - 27-14-0 broadcast

#### 3.8.3.5 Miscellaneous Data.

As well as the major elements listed above, the following data on minor farm costs were required.

- a) Labor cost - \$1.00/hour
- b) Freight on livestock - \$ .80/100 lb.
- c) Fuel cost - diesel = \$ .23/Imperial gallon  
- gasoline = \$ .27/Imperial gallon







- d) Seed cost - wheat - \$3.00/acre
  - barley - \$2.25/acre
  - clover - \$3.00/acre
  - grass - \$3.00/acre
  - flax - \$3.00/acre
  - rape - \$ .50/acre

e) Baling cost (custom) - \$ .10/bale, 40 bale/ton

- f) Fertilizer cost - 11-48-0 \$107.00/ton
  - 27-14-0 \$ 89.00/ton
  - 16-20-0 \$ 75.00/ton

g) Spray \$26.50/5 gallons 2,4D Ester (128 oz. acid/gallon)

#### 3.8.4 Results.

Gross Margin - \$11,330.00

#### Cropping Program

- Buy an additional 155 acres.
- Creeping Red Fescue - 276 acres  
(4 years in grass before reseeding)
- Barley - 10.4 acres on stubble
  - 5.2 acres on fallow  
(three years grain, one year fallow)
- LaSalle clover - 81 acres  
(three years clover, one year fallow)
- Fallow - 32.2 acres  
(5.2 for barley, 27 for clover)

#### Livestock Program

- Sell weanlings from 12 sows
- Feed home grown barley plus supplement.

#### Miscellaneous

- Pick up 6.25 tons of straw for bedding.

#### 3.8.5 Remarks.

Two assumptions are questionable in this case study. They are:



- a) That no upper limit needs to be placed on the amount of roughage consumed by cows when all other classes have limits.
- b) That the yields of forage seed may be averaged without consideration for the length of time since the crop had been seeded.

The preparation of the linear program for this farmer was started on September 30, 1964 and was ready to punch on cards on January 4, 1965. Data preparation, according to the format specified by Clasen<sup>13</sup>, required one person for 8.5 hours. Two trials were necessary before the solution was obtained. The results were sent to the District Agriculturist on February 5, 1965 and no requests were made for re-runs.

### 3.9 CASE 9

#### 3.9.1 Resources.

The resources of this farmer are as follows:

- a) Land - arable - 425 acres  
- possible breaking - 75 acres
- b) Livestock resources:
  - Swine space - 18 sows and litters
  - Swine herd (present) - 14 sows and litters
  - Cow herd - 65 cows
  - Minimum cow herd - 40 cows
  - Feedlot space - 90 yearling steers at once.
- c) Capital - Operating - a) \$ 6,500.00 b) Unlimited  
- Livestock - \$10,000.00  
- Real Estate - \$ 5,000.00
- d) Labor available - Operator plus son year around.

#### 3.9.2 Farmer's Preferences.

This farmer would consider only wheat, barley, oats, or hay for crops as he felt these were the best suited to his land. Also any feasible beef or swine program would be considered except that he



wanted a minimum cow herd of 40 head.

### 3.9.3 Data Inputs.

#### 3.9.3.1 Crop Yields and Prices.

The crop yields were estimated by the farmer, and the grain prices based on his records and the prices listed by elevator companies. The prices used in this program are given in Table 32.

Table 32. Crop Yields and Prices for Case 9.

Crop	Yield		Farm Price	
	On Plowed Hay Land	On Stubble	Sold	Purchased
Barley	52 bus/acre	42 bus/acre	\$ .80/bu	\$34.28/ton
Oats	80 bus/acre	65 bus/acre	\$ .50/bu	\$31.00/ton
Wheat	40 bus/acre		\$ 1.30/bu	
Hay		2 ton/acre	\$10.00/ton	\$11.00/ton

#### 3.9.3.2 Field Machinery.

Table 33 contains the equipment data used in Case 9.



Table 33. Basic Machinery Data Used in Case 9.

Machine	Size or Model	Original Price
Tractors	I.H.C. 560 Diesel	\$ 7,300.00
	Farmall H Gasoline	\$ 3,000.00
Cultivator	12 ft.	\$ 800.00
Plow	5.3 ft (4-16")	\$ 550.00
Baler	New Holland 69 (8 tons/hr.)	\$ 1,800.00
Mower	9 ft.	\$ 900.00
Rake	9 ft.	\$ 500.00
Drill (Double disc, grass seeder)	12 ft.	\$ 2,200.00
Swather	12 ft.	\$ 1,200.00
Combine	John Deere 95	\$10,000.00
Hammermill	14 inch, moveable	

### 3.9.3.3 Livestock.

#### 3.9.3.3.1 Beef cattle:

The data for beef cattle were the same as for Case 8 with the exception that the transportation costs were reduced to 50% of the value in Case 8.

#### 3.9.3.3.2 Swine:

Relatively complete data are given here since these data were used in subsequent studies. The data are as follows:

##### - Time:

25 sows and litters require 1.5 hour per day year around.

Estimated time for registered stock - 1/3 of litter requires 1.5 times as much time as fattening hogs.



- Feed:

The feeding data are based on the farmer's record of 7.0 months to fatten a market hog and his rations which are as follows:

a) Sow:

- total feed - 2,500 lb/sow
- 23.1% barley, 69.4% oats, 7.5% of 35% sow supplement
- litter weaned at 8 weeks or 30 lb/weanling
- 8 pigs per litter and 2 litters per year.

b) Fattening hogs:

- Feed requirements from N.R.C.<sup>41</sup>
- 30 lb - 70 lb 2.6 lb/day
- 70 lb - 150 lb 6.0 lb/day
- 150 lb - 200 lb 6.5 lb/day

- Feed used:

The farmer's ration was used for proportions.

Total requirements as computed were checked with L.W. Mellemstrand<sup>37</sup>.

- 30 lb - 150 lb 514 lb/pig  
15.0% of 42% supplement, 63.8% barley, 21.2% oats
- 150 lb - 200 lb 241 lb/pig  
7.5% of 42% supplement, 69.4% barley, 23.1% oats
- Supplement cost - 35% sow \$120.00/ton
- 42% hog \$120.00/ton
- Grinding feed \$ .40/ton
- Sow capital value - Grade herd \$ 80.00/sow
- Registered herd \$150.00/sow

- Operating costs:

The supplement costs are only variable feed costs which are considered directly. Grain costs vary, since the grain may be raised or purchased. The costs for the classes are as follows:

a) Sow - Supplement	\$ 11.40
- Grinding - 2,310 lb grain	\$ 1.15
- Miscellaneous	<u>\$ 15.00</u>
	\$ 27.55/year



b) Hogs - Supplement	\$ 91.44
- Grinding - 10,556 lb grain	\$ 2.11
- Miscellaneous	<u>\$ 15.00</u>
	\$108.55/year

- Returns:

The basis here is the gross value at the farm gate less the feeding costs above. The following data were used in this program.

a) Fat hogs - 150 lb/hog dressed weight	
- \$23.00/100 lb dressed weight	
- \$34.50/hog	
- shipping and selling cost/hog	\$ 1.00
- gross return on farm/hog	\$ 33.50
- gross return on farm/16 hogs	\$536.00/sow
- activity gross margin	
(\$536.00 - \$108.55 - \$27.55)	\$399.90

This was rounded off to \$400.00 per sow.

b) Selling weanling:	
- Gross return on the farm/pig	\$ 9.00
	\$144.00/sow
- activity gross margin	
(\$144.00 - \$27.55)	\$116.45

c) Fatten weanling pigs - 16 pigs/unit	
- Purchase at \$9.50/30 lb hog	
- Sell at 200 lb for \$33.50/pig on the farm	
- Variable costs (supplement, miscellaneous)	\$108.55
- Activity gross margin	
(\$536.00 - \$152.00 - \$108.55)	\$275.45/16 pigs

#### 3.9.3.4 Farming Practice.

This farmer wanted at least one half of his land in forage crops. Land to be seeded to grain was cultivated in May prior to seeding with the drill. The crop was then left to late August for swathing and then combined in September. All this land was then fall cultivated. Grass land was plowed in August after the hay had been removed and received two



workings with the cultivator before freeze-up. The fertilizer practice was 100 lb/acre of 16-20-0 on grain crops put on at seeding, and 80 lb/acre of 21-0-0 broadcast on the grass land. As well, the farmer did not want land seeded to the same grain crop two years in a row.

### 3.9.3.5 Miscellaneous Data.

As well as the above data, the following costs of supplies were used:

- a) Fuel cost - diesel - \$ .20/Imperial gallon  
- gasoline - \$ .23/Imperial gallon
- b) Seed cost - Oats - \$1.50/acre  
- Barley - \$2.00/acre  
- Wheat - \$2.50/acre  
- Grass - \$3.00/acre
- c) Fertilizer cost - 16-20-0 \$72.00/ton  
- 21-0-0 \$51.00/ton
- d) 32% beef supplement - \$90.00/ton
- e) Labor - \$1.00/hour
- f) Baler twine - \$ .02/bale, 40 bales/ton
- g) Breaking land - \$25.00/acre
- h) New feedlot - \$23.40/yearling capacity added.

Basis: 100 head of yearlings requiring 30,000 ft<sup>2</sup> of space with the following equipment:

- Fence, 1,000 ft	\$1,240.00
- Shed	\$ 450.00
- Bin	\$ 450.00
- Water supply	<u>\$1,100.00</u>
	\$3,240.00

This was from the Grain Grower<sup>24</sup> with the exception of the cost of the water supply, which was the farmer's estimate.



[illegible]

Figure 12. Linear Programming Matrix Prepared for Case 9.

\* This row was removed on the second run.



#### 3.9.4 Results.

Two runs were done on this farm. One was with a \$6,500.00 restriction on operating capital and the other with no restriction. The results are as follows:

##### 1) \$6,500.00 restriction.

Gross Margin - \$17,200.00

##### Cropping Program

- Break 75 acres
- Wheat on grass - 83 acres
- Barley on stubble - 83 acres
- Oats on stubble - 83 acres
- Hay - 170 acres
- Pasture - 80 acres

##### Livestock Program

- Sell weanlings from 61 sows
- Sell calves from 40 cows
- Fatten 46 long yearling steers over winter
- Feed:

Hogs - 53 tons oats  
- 17 tons barley  
- 5 tons 35% sow supplement

Beef - Winter: 39 ton oats  
39 ton barley  
46 ton hay  
- Summer: 80 acres pasture for cows

##### Miscellaneous

- Sell 26 tons of barley, 294 tons hay, 25 cows
- Pick up 72 tons straw for bedding
- Add 600 ft<sup>2</sup> swine barn for \$3,125.00



2) No restriction on operating capital.

Gross Margin - \$20,000.00

Cropping Program

- Break 75 acres
- Wheat, barley and oat acreages as above  
(wheat sold, barley and oats fed to own stock)
- Hay - 155 acres
- Pasture - 95 acres  
(80 for cows, 15 for yearlings)

Livestock Program

- Fatten and sell 976 weanling pigs per year
- Sell calves from 40 cows
- Fatten and sell 90 long yearling steers over winter
- Fatten and sell 53 yearling steers over summer
- Feed:

Hogs - 80 ton oats  
- 241 ton barley  
- 46 ton 42% hog supplement

Beef - Winter: 124 ton oats  
32 ton straw  
58 ton hay  
- Summer: 43 ton oats  
22 ton hay  
95 acres pasture

Miscellaneous

- Sell all sows and 25 cows
- Sell 230 tons hay every year
- Buy 155 tons oats, 138 tons barley
- Pick up 250 tons of straw for bedding



### 3.9.5 Remarks.

Two assumptions in this case study may be questioned. They are:

- a) that the depreciation allowance for the feedlot and swine building is not linear so it should be left out of the activity costs.
- b) that an upper limit on the amount of roughage for cows was not required when it was placed on all other stock.

This program has the major improvement in the data used for the swine in using a longer period of fattening.

This program was begun on January 19, 1965. The data preparation for the computer was done on February 9, 1965 requiring one person 3.0 hours to punch the cards and two people .5 hours to check the data. The problem required five trials before the solution was satisfactory. Each time checking required about 2.5 hours to check through the program. The program was finished March 2, 1965 and sent to the District Agriculturist in the area. No changes were requested.

## 3.10 CASE 10

### 3.10.1 Resources.

The farmer had the following resources at his disposal:

- a) Arable land - 800 acres
- b) Swine barn capacity - 45 sows and litters
- c) Capital - Operating - \$13,000.00  
- Livestock - \$ 8,000.00  
- Real Estate - \$10,000.00
- d) Labor - Operator plus family labor equivalent to two men full time plus an additional man from July to October.

### 3.10.2 Farmer's Preferences.

This farmer has a slight preference for hogs and grain, but would



consider almost any type of enterprise which had proven feasible in the area.

### 3.10.3 Data Input.

#### 3.10.3.1 Crop Yields and Prices.

The following yields and prices were based on the farmer's estimate and records for grain and C.R. Elliot<sup>18</sup> for forage crops.

- a) Wheat - \$1.40/bushel - 33 bushels/acre
- b) Barley - \$ .80/bushel (sold) - 45 bushels/acre  
- \$34.58/ton (purchased and delivered)
- c) Oats - \$ .51/bushel (sold) - 50 bushels/acre  
- \$32.28/ton (purchased and delivered)
- d) Forage seed\*:
  - Alsike clover seed - \$ .13/lb  
First year after seeding - 250 lb/acre  
Second year after seeding - 300 lb/acre
  - Altaswede red clover seed - \$ .17/lb commercial seed  
- \$ .22/lb if foundation stock  
First year after seeding - 175 lb/acre  
Second year after seeding - 200 lb/acre
  - Creeping red fescue - \$ .20/lb commercial seed  
1.2 ton hay aftermath  
- \$ .25/lb foundation stock  
300 lb/acre
- e) Hay - Grass - \$10.50/ton (sold) - 1.9 ton/acre  
- \$11.00/ton (purchased and delivered)
- Clover - \$11.50/ton (sold)  
- \$12.00/ton (purchased and delivered)  
First year after seeding - 1.5 ton/acre  
Second year after seeding - 1.75 ton/acre

\* Prices were a trade estimate.



### 3.10.3.2 Field Machinery.

The farmer's machinery is given in Table 34.

Table 34. Basic Farm Machinery Data for Case 10.

Machine	Size or Model	Original Cost
Tractor	John Deere 730 Propane	\$ 7,000.00
Cultivator	12 ft.	\$ 800.00
Double Disc	12 ft.	\$ 1,125.00
Plow	4 ft. (3-16")	\$ 850.00
Press Drill	14 ft.	\$ 2,400.00
Harrow	36 ft.	\$ 500.00
Sprayer	32 ft.	\$ 500.00
Swather (self-propelled)	12 ft.	\$ 3,000.00
Combine (self-propelled)	503 International	\$12,000.00

### 3.10.3.3 Livestock.

The data for beef cattle were taken directly from Case 8, while the swine data came from Case 9 with some modifications. These modifications were as follows:

- a) No program of feeding weanlings was considered.
- b) The on-farm gross return per hog was reduced to \$32.75 due to increased transportation costs to market.
- c) The farmer had the opportunity to sell breeding stock if he improved the quality of his herd. The costs were the same as for fattening hogs, but the time requirement increased by 50% for 1/3 of the offspring and the returns were changed to those shown below.
  - Sell 1/3 of herd for breeding stock.
  - Sell 1/3 of litter at \$15.00 above market price.  
(farmer's estimate)



- Returns per sow per year (price at the farm)  
 $1/3 \times 16 \times (32.75 + 15.00) + 2/3 \times 16 \times 32.75 =$   
 $256.72 + 363.33 = \$620.05$
- Margin to feed, labor and overhead  
 $\$620.05 - \$144.57 = \$475.48$

#### 3.10.3.4 Farming Practices.

This operator maintained a rotation without summerfallow. Clover crops were seeded with the previous grain crop. Clover land to be broken up was worked with the plow followed by the double disc, with harrows in tandem after the clover crop was harvested. Grain land was worked with the cultivator and harrows in tandem in early May. The land was then worked again with the harrows just prior to seeding with a hoe drill in mid-May. The crop was then sprayed in June, swathed in late August and combined in September. The farmer spread 33.5-0-0 at the rate of 100 lb/acre on all land except clover, being plowed up. All land seeded to grain then received 11-48-0 at the rate of 40 lb/acre in the spring. Hay was swathed and custom baled in June and July. Fescue aftermath received the same procedure in September.

#### 3.10.3.5 Miscellaneous Data.

The following miscellaneous data were required.

- a) Fuel cost - propane - \$ .13/Imperial gallon  
- gasoline - \$ .24/Imperial gallon
- b) Seed cost - Clover - \$3.00/acre  
- Grass - \$3.00/acre  
- Wheat - \$2.00/acre  
- Barley - \$2.00/acre  
- Oats - \$1.50/acre  
Foundation stock - \$1.00/acre for contract
- c) Fertilizer cost - 33.5-0-0 - \$ 75.00/ton  
- 11-48-0 - \$100.00/ton



[illegible]

Figure 13. Linear Programming Matrix Prepared for Case 18.



- d) 32% beef supplement - \$100.00/ton
- e) Labor cost \$1.00/hour
- f) Baler (custom rate) - \$ .10 /bale, 40 bales/ton
- g) New feedlot space - \$24.40/yearling  
(This is cheaper than Case 9 since the water supply was \$800.00 cheaper).

#### 3.10.4 Results.

Gross Margin - \$52,200.00

#### Cropping Program

- 800 acres in 1 year grain, 3 years grass with the following crops:
  - 100 acres barley
  - 100 acres oats
  - 150 acres registered fescue
  - 450 acres commercial fescue

#### Livestock Program

- Hogs - sell weanlings from 75 sows
  - sell breeding stock from 13 sows

#### Miscellaneous

- Add 493 ft<sup>2</sup> to hog barn (cost = \$1,450.00)
- Sell 29.4 tons barley (not needed for swine)
- Buy 9.2 tons oats

#### 3.10.5 Remarks.

This program was improved by entering the yields of forage seed and hay according to the length of time the crop had been seeded to grass. The program was started on February 12, 1965 and worked on exclusively until it was completed on February 17, 1965. The card punching required one person two hours and checking required one person one hour. Three trials were required before the solution was obtained on March 5, 1965 and sent to the District Agriculturist. There were no requests for re-runs.



### 3.11 CASE 11

#### 3.11.1 Resources.

The farmer had the following resources available:

- a) Land - arable - 410 acres (250 suitable for grain or hay,  
160 suitable only for hay)
  - pasture - 3,200 acres
- b) Cows - 130 head
- c) Capital - Operating - \$20,000.00
  - Livestock - \$20,000.00
  - Equipment - \$10,000.00
- d) Labor supply - Operator

#### 3.11.2 Farmer's Preferences.

This was a ranching operation and the operator was interested only in the best way to utilize the roughage through beef. Also he did not want to expand the land any more. His arable land rotation was fixed in grain, fallow or grass reseeded every five years.

#### 3.11.3 Data Inputs.

##### 3.11.3.1 Crop Yields and Prices.

Table 35 gives the crop data used in Case 11.

Table 35. Crop Yields and Prices Used in Case 11.

Crop	Yield	Farm Price	
		Sold	Purchased
Wheat	20 bus/acre	\$ 1.40/bu	
Oats	40 bus/acre	\$ .55/bu	\$40.00/ton
Barley	30 bus/acre	\$ .75/bu	\$45.00/ton
Hay	.5 ton/acre	\$20.00/ton	\$25.00/ton



### 3.11.3.2 Field Machinery.

Table 36 contains the list of equipment used in this case study. In addition, this operator had the option of buying a sprayer, a swather and a combine or hiring custom operators. The custom rates were as follows:

Spraying - \$ .70/acre

Swathing - \$ 1.00/acre

Combining - \$ 4.00/acre

Since the operator would have to pay operating costs regardless of the method used, the alternative procedures were compared on the lowest capital cost per year based on MacHardy's<sup>34</sup> procedure for including the equipment. The cost of owning a machine was based on a five year pay-off period for depreciation and these were as follows:

Sprayer - \$ 6.00/year/acre per hour capacity  
10 hours available for spraying.

Swather - \$ 52.00/year/acre per hour capacity  
40 hours available for swathing.

Combine - \$217.00/year/acre per hour capacity  
50 hours available for combining.



Table 36. Basic Equipment Data Used in Case 11

Machine	Size or Model	Original Cost
Tractor	Massey Ferguson 65 Diesel	\$4,750.00
Cultivator	9 ft.	\$ 450.00
Tandem Disc	8 ft.	\$ 400.00
Hoe Drill	10 ft.	\$2,000.00
Harrow	30 ft.	\$ 150.00
Mower	8 ft.	\$ 500.00
Rake	8 ft.	\$ 500.00
Baler	Massey Ferguson No. 10	\$1,600.00

#### 3.11.3.3 Livestock.

The nutrient requirements were the same as calculated for Case 5. The labor requirements were obtained from Case 10 with some modifications which were from the rancher. The activities and the economic data were obtained from the rancher. A summary of these data appear in Tables 37 and 38.

#### 3.11.3.4 Farming Practices.

This operator worked his arable land in fallow four times during the season; cultivate in May, July and September, and disc in June. Land seeded to grain was cultivated twice and disced once prior to seeding with a hoe press drill in May. At seeding, 50 lb/acre of 11-48-0 was applied on grain. Then the crop was sprayed in June, swathed in August and combined in September. Hay was mowed, raked, and baled in June and July. All grass land received 200 lb/acre of 16-20-0 in the spring.



Table 37. Data on the Alternative Methods of Utilizing the Cow Herd in Case 11.

Product	Units		Selling		Operating Costs Except Feed	Activity Gross Margin
	Herd	Sale	Price (\$/lb)	Weight (lb)		
Sell calves (bulls used non-creep fed)	1 cow	.45 steer	.275	400		
	.04 bull	.30 heifer	.250	375		
	.15 replacement heifer	.14 culls	.140	900	\$11.95	\$90.15
Sell calves (bulls used creep fed)	1 cow	.45 steer	.275	450		
	.04 bull	.30 heifer	.250	425		
	.15 replacement	.14 culls	.140	900	\$11.95	\$108.08
Sell calves (Charolais A.I. non-creep fed)	1 cow	.45 steer	.250	500		
	.15 replacement	.30 heifer	.225	475		
		.14 culls	.140	900	\$18.13	\$ 87.82
Sell calves (Charolais A.I. creep fed)	1 cow	.45 steer	.250	600		
	.15 replacement	.30 heifer	.225	575		
		.14 culls	.140	900	\$18.13	\$105.82
Sell long yearlings	1 cow	.445 steer	.225	750		
	.04 bull	.297 heifer	.200	675		
	.15 replacement	.14 culls	.140	900	\$15.13	\$117.70
Sell 2 year old bred heifer, long yearling steer	1 cow	.445 steer	.225	750		
	.04 bull	.294 heifer	.230	975		
	.15 replacement	.14 culls	.140	900	\$16.23	\$142.43
Sell 2 year old stock	1 cow	.441 steer	.240	1,050		
	.04 bull	.294 heifer	.230	975		
	.15 replacement	.14 culls	.140	900	\$17.90	\$176.80



Table 38. Summary of the Partial Budgets for Fattening Stock in Case 11.

Class	Unit	Buying		Selling		Operating Costs Except Feed	Activity Gross Margin
		Price (\$/lb)	Weight (lb)	Price (\$/lb)	Weight (lb)		
Fatten steer calves	calf	.275	450	.250	900	\$15.20	\$86.05
Fatten heifer calves	calf	.250	425	.230	825	\$12.40	\$71.10
Grow steer calves and fatten in summer	steer	.275	450	.250	950	\$14.00	\$99.25
Grow heifers and fatten in summer	heifer	.250	425	.230	850	\$11.60	\$77.65
Fatten yearling steers	steer	.250	600	.250	950	\$14.45	\$73.05
Fatten yearling heifers	heifer	.230	550	.230	850	\$11.75	\$57.25



[illegible]

Figure 14. Linear Programming Matrix Prepared for Case 11.



### 3.11.3.5 Miscellaneous Data.

The following costs of farm supplies were required.

- a) Fuel cost - diesel - \$ .185/Imperial gallon  
- gasoline - \$ .22/Imperial gallon
- b) Seed cost - Wheat - \$3.00/acre  
- Barley - \$2.25/acre  
- Grass - \$3.00/acre
- c) Fertilizer cost - 16-20-0 - \$ 75.00/ton  
11-48-0 - \$100.00/ton
- d) Livestock cost - artificial insemination - \$ 8.00/cow  
- feedlot space - capital cost - \$60.00/yearling  
- depreciation - \$12.00/yearling  
per year
- e) Labor - \$1.00/hour.

The feedlot cost included the cost of feed handling equipment. Other farms already had this equipment and the cost was correspondingly lower.

### 3.11.4 Results.

Gross Margin - \$19,985.00

#### Cropping Program

- 84 acres barley
- 41 acres oats
- 125 acres fallow

#### Livestock

- 98 cows - sell 2 year old feeder steers and 2 year old bred heifers for herd replacement in other herds.
- 59 cows - sell creep fed calves from Charolais and Hereford cross.
- Buy 27 cows to bring herd to 157 cows.
- Fatten - 22 steer calves in winter  
- 74 yearling steers in summer (22 in lot, 52 on grass)



- Feed: Winter - 210 ton hay
  - 197 ton straw
  - 154 ton barley
  - 627 acres winter pasture
- Summer - (cows are out on pasture, this feed is for steers)
  - 105 acres native grass (to animals on grass)
  - 22 ton oats
  - 39 ton barley
  - 20.6 ton hay (to animals in lot)

#### Miscellaneous

- Build feedlot to hold 22 yearling steers
- Bale 104 ton of straw
- Buy - 120 ton of straw
  - 151 ton of hay
  - sprayer with 12.5 acres per hour capacity
  - swather with 3.1 acres per hour capacity
  - combine with 2.5 acres per hour capacity
- Hire one man year around plus casual labor in April.

#### 3.11.5 Remarks.

This operator provided a detailed estimate of data for all the livestock activities that he would like to consider. Thus the cost of replacement heifers and bulls for the cow herd was not simply a depreciation allowance but was integrated into the total feed and pasture requirements. Also the operator estimated the effects of such practices as creep feeding, Charolais x Hereford crossbreds, and being able to sell breeding stock. As well, a restriction was added for maximum intake of roughage by the cows and an entry for the depreciation of new feedlots was made.

The program might have been improved further by including the following considerations:

- a) Another rotation.
- b) Adjusting the selling price of barley which is abnormally low.



This program was begun on July 14, 1964 and worked on exclusively until it was finished. Preparation of the original matrix required one experienced person and one person who was learning the technique for sixteen hours. In addition the experienced person spent another thirteen hours in putting the final touches to the program. Coding required one person for three hours on July 20, 1965. Punching required 3.7 hours and checking required 2.0 hours. Three trials were required before the final results were obtained. The first trial, (finished on July 21, 1965), was unsatisfactory due to a punching error. The second trial (July 22, 1965) was required to correct this while the third trial (August 5, 1965) was required due to a change in the roughage restriction on the cattle. A re-run (August 9, 1965) was requested by the operator due to a misunderstanding on available arable land. Organizing the data to a form easily understood by the operator (August 11, 1965) required two hours and a discussion of the results with the operator (August 12, 1965) required 2.5 hours. Each time checking for a re-run required three hours. The problem was solved by August 9, 1965, but the periods of July 16, 1965 to July 18, 1965, and July 26, 1965 to August 5, 1965 represent time not spent on the problem. Thus the problem required forty-three man hours to put in matrix form, and three man hours to scale which was done in a period of four working days. Periodic work on the program took place until August 9, 1965. Computer time on this problem was only 1.8 to 2.5 minutes per trial, but there was time in waiting for the problem to be run on the computer. This time was a minimum of one-half day and usually about one day.



### 3.12 CASE 12

This problem was prepared by Dr. F.V. MacHardy, but data from previous programs were used for much of this program. The problem processing and analysis was done by the author.

#### 3.12.1 Resources.

The farmer had the following resources:

- a) Land - Arable
  - high land 410 acres
  - low land 280 acres
  - Possible new breaking
    - low land 80 acres
  - Rough low (includes possible land pasture breaking) 110 acres
- b) Cows 48 head
- c) Capital \$10,000.00

#### 3.12.2 Farmer's Preferences.

This farmer had a distinct preference for a beef cow enterprise. The farmer and the District Agriculturist of the area both felt that forage crops had to be on at least one half of the high land and on at least one third of the low land. Also, this farmer had no land suitable for wheat and he wished to maintain a rotation of barley, barley, oats on his grain land.

#### 3.12.3 Data Inputs.

##### 3.12.3.1 Crop Yields and Prices.

The farmer supplied most of the following data:



a) Barley     \$ .80/bushel

High land - wet year	40 bushels/acre
- dry year	20 bushels/acre

Low land - wet year	
(seed only half	
of acreage due	
to too wet in	
spring)	20 bushels/acre
- dry year	40 bushels/acre

b) Oats         \$ .55/bushel

High land - wet year	56.5 bushels/acre
- dry year	30 bushels/acre

Low land - wet year	28 bushels/acre
- dry year	56.5 bushels/acre

c) Silage (forage crops)

High land - wet year	5 ton/acre
- dry year	2 ton/acre

Low land - any year	5 ton/acre
---------------------	------------

3.12.3.2 Field Machinery.

The farmer gave a list of his machinery which is given in Table 39.



Table 39. Machinery Available to Farmer in Case 12.

Machine	Size or Model
Tractors	Allis Chalmers D17 Diesel Allis Chalmers WD Diesel International W6 Gasoline Caterpillar D4 Diesel
Cultivator	15 ft.
Tandem Disc	10 ft.
Harrows	30 ft.
Drill (Grass Seeder, Fertilizer)	10 ft.
John Deere forage harvester	5 ft.
Baler	New Holland 69 Hayliner
Plow	5.3 ft. (4-16")
Swather	12 ft.
Combine (self-propelled)	Oliver 33 (12 ft. table)
3 Racks and wagons	
Hammermill	200 bus/hour

#### 3.12.3.3 Livestock.

Only beef cattle activities were considered in this program and the data came from Case 3 with the exception of calf price. This price was reduced to \$23.00/100 lb. from \$26.00/100 lb.

#### 3.12.3.4 Farming Practice.

No listing was given by the farmer, so it was assumed that he followed a sequence of: cultivate early in May, disc followed by drill







and harrow in mid-May, swath in August, combine in September, and plow in late September, unless land was seeded to forage. Forage was assumed to be seeded with the previous oat crop. Forage land to be broken up was assumed to be plowed in August and disced twice in September and October. The farmer mentioned his fertilizer practice as being broadcasting 170 lb. per acre of 21-0-0 in the fall on the land that was being fall plowed, and 30 lb. per acre of 21-0-0 in the spring with the grain. If grain is the first crop after hay, the spring application is increased to 85 lb. per acre of 21-0-0.

#### 3.12.4 Results.

The program was initially run with only the first 23 restrictions for both wet and dry years. The results were:

a) Wet year:

Gross Margin      \$14,162.00

#### Cropping Program

- Sell all barley and oats produced.
- High land - 205 acres forage used for silage
  - 205 acres grain, 2/3 barley, 1/3 oats
- Low land - 93 acres grass used for silage
  - 187 acres grain, 2/3 barley, 1/3 oats

#### Livestock

- Sell the 48 cows
- Fatten 185 calves over winter
- Fatten 8 yearlings over summer
- Feed:
  - Summer - 110 acres rough pasture
  - Winter - 1491 ton silage
    - 6.75 ton 32% supplement



b) Dry year:

Gross Margin        \$12,659.00

Cropping Program

- Sell barley and surplus oats
- High land - 221 acres grain, 2/3 barley, 1/3 oats
  - 189 acres grass - grazing
- Low land - Break the additional 80 acres
  - 120 acres grass for silage
  - 240 acres grain, 2/3 barley, 1/3 oats

Livestock

- Sell all the cows
- Fatten 100 calves over winter
- Fatten 51 yearlings over summer
- Feed:

Summer - pasture  
Winter - 600 tons silage  
          - 2135 bushels oats  
          - 30 tons straw

It was left to the farmer to choose which was likely to be the best program for the year at hand. These results were sent to the District Agriculturist on May 4, 1964. A reply was received from the assistant District Agriculturist on July 25, 1964 to request an explanation of the winter ration for a wet year. The results had been sent out with an error in terms. Instead of 135 - 100 lb. units (6.75 tons) of 32% supplement, the results were sent as 135 tons of 32% supplement. This was immediately corrected in a letter to the assistant District Agriculturist on July 27, 1964, but unfortunately the personnel was in a state of change at the District Agriculturist's office in the area. As a result, this letter was filed without the farmer seeing the correction. The consequence



was a good deal of dissatisfaction on the part of the farmer, which he communicated to a number of farmers in the area. This reached the Department of Agricultural Engineering late in January of 1965 and was followed up by a visit by Dr. F.V. MacHardy in March. Subsequently another two runs were done on this program and the results sent out on March 23, 1965. Two different capitalization levels were used, the restrictions 24, 25, and 26 were added, and the cow herd on hand was increased to 90 head. The results with the capital limit at \$10,000.00 are as follows:

a) Wet year:

Gross Margin            \$10,574.00

Cropping Program

- High land - 205 acres forage, (180 acres for grazing  
25 acres left idle)
- 205 acres grain, 2/3 barley, 1/3 oats
- Low land - 187 acres grain, 2/3 barley, 1/3 oats
- 97 acres grass, 69 acres silage,  
34 acres idle)

Livestock

- Raise calves from 90 cows.
- Fatten 81 calves raised plus 8 head purchased  
to bring the total to 89 head
- Feed:
  - 296 tons silage
  - 4159 bushels of barley
  - 5603 bushels of oats
- Sell 3,800 bushels of barley

b) Dry year:

Gross Margin            \$10,357.00



Cropping Program

- High land - 205 acres grain, 2/3 barley, 1/3 oats
  - 205 acres grass, (146 acres grazing, 59 acres idle)
- Low land - 187 acres grain, 2/3 barley, 1/3 oats
  - 93 acres grass, (59 acres silage, 34 acres grazing).

Livestock

- Numbers same as wet year.
- Feed - 296 tons silage
  - 4186 bushels barley
  - 5566 bushels oats
- Sell 3525 bushels barley

The results with the capital limit at \$15,000.00 are as follows:

a) Wet year:

Gross Margin      \$11,834.00

Cropping Program

- Acreages as for wet year above.
- Use - 180 acres of grass on high land for grazing
  - 89 acres of low land grass for silage
  - 29 acres of grass is left idle

Livestock

- 90 cows raise 81 calves
- Fatten these plus 52 head purchased
- Feed:
  - Winter - 444 tons silage
  - 6028 bushels barley
  - 5603 bushels oats
- Sell 1,927 bushels barley

b) Dry year:

Gross Margin      \$11,616.00



Cropping Program and Livestock Program

- Same as wet year above.
- Feed:
  - 444 tonsilage
  - 6055 bushels barley
  - 5365 bushels oats
- Sell 1,655 bushels oats

3.12.5 Remarks.

This program might have been improved by

- a) Separating the fattening animals according to sex.
- b) Putting a value in the roughage restriction on the livestock for cows to allow cows to eat as much roughage as possible.

The program with the first 23 restrictions was prepared by Dr. F.V. MacHardy and was ready for punching on April 9, 1964. Punching cards required .75 hours for two people and checking required .5 hours for one person. This problem was initially run on the 1620 computer and required three trials to complete. Checking time for each trial was 2.5 hours with the final trial being completed on May 4, 1964. The re-runs done in March of 1965 were done on the 7040 computer and required four trials to complete.



## 4. ANALYSIS

### 4.1 TIME SPENT ON PROGRAMS

Unfortunately there was not a detailed record kept of the number of hours taken in actual program preparation and solving. For this reason the time must be estimated to a certain degree if a true comparison is desired. This is not as bad as it sounds as the procedure of working on only one problem at a time was followed. The usual case was to begin another case study as soon as the previous study had been handed into the computing centre for the first trial. The final date of preparing the program for the initial trial was recorded so the date of beginning another program could be accurately established. There were exceptions to this procedure, but they were well marked.

Another adjustment must be made to the time spent on these problems. Half of the case studies were done while the programmer had other commitments. While accurate data are not available, it is estimated that 25% of the total time in these periods was spent actually working on the program. Actual working time per week was approximately 40 hours so approximately 10 hours per week was spent on linear programming problems.

The other six programs were prepared by working exclusively on them. The working time per week was approximately 40 hours. All these data are shown in Table 40 and Table 41.



Table 40. Pertinent Dates in Preparing and Solving the Case Studies.

Case No.	Start Matrix	Matrix Complete	Solution Obtained	Re-Runs	Computer
1	Oct. 15, 1963	Dec. 17, 1963	Jan. 7, 1964	1	IBM 1620*
2	Dec. 15, 1963	Jan. 27, 1964	Feb. 20, 1964	1	IBM 1620*
3	Feb. 1, 1964	Mar. 8, 1964	Apr. 2, 1964	0	IBM 1620**
4	June 2, 1964	June 11, 1964	Oct. 7, 1964	0	IBM 1620***
5	June 13, 1964	July 12, 1964	July 30, 1964	0	IBM 1620***
6	July 14, 1964	July 21, 1964	Sept. 8, 1964	0	IBM 1620***
7	July 22, 1964	July 27, 1964 (Feb. 9, 1965)	Oct. 6, 1964 (Last trial on IBM 1620***) (Mar. 10, 1965)	1	IBM 7040
8	Sept. 30, 1964	Jan. 3, 1965	Feb. 5, 1965	0	IBM 7040
9	Feb. 2, 1965	Feb. 8, 1965	Mar. 2, 1965	0	IBM 7040
10	Jan. 19, 1965	Feb. 17, 1965	Mar. 5, 1965	0	IBM 7040
11	July 14, 1965	July 20, 1965	Aug. 9, 1965	1	IBM 7040
12		Apr. 9, 1964	May 4, 1964	1	IBM 1620***

\* The program written by Wood<sup>52</sup> was used for the first solution and the program by Nichols and Krieger<sup>39A</sup> was used for the re-run.

\*\* The program written by Wood<sup>53</sup> was used.

\*\*\* The program written by Nichols, Nickel and Davis<sup>39B</sup> was used.



Table 41. Estimated Time Spent on Preparing the Matrix and Solving the Problem.

Case No.	Recorded Time to Complete Matrix (Weeks)	Estimated Actual Programming Time (Weeks)	Recorded Time to Obtain Solution (Weeks)	Number of Trials for Solution
1	9.1	2.3	3.0	4
2	6.3	1.6	3.4	3
3	5.1	1.3	3.4	3
4	1.4*(5.6)	1.4	13.7**(9.7)	5
5	4.3	1.1	2.4	4
6	1.1*(4.4)	1.1	6.4**(2.4)	2
7	.9*(3.6)	.9	10.1**(6.1) 4.3 7040	1620 3 7040 5
8	13.6*** (4.6)	1.2	4.4	2
9	1.0*(4.0)	1.0	3.0	5
10	4.1**** (3.1)	.8	2.1	3
11	1.0*(4.0)	1.0	2.9	3
12			3.4	3

\* The programmer worked exclusively on the matrix for this period.

\*\* Deduct four weeks for August when 25% of the month was not spent on programming.

\*\*\* Deduct nine weeks (mid-October to mid-December) for time waiting for the 7040 computer and checking for a satisfactory computer program.

\*\*\*\* Deduct one week for work on Case 9.  
The programs not marked with footnotes were prepared while working on other commitments. The figures in brackets are the corrected times for reasons given in the footnotes.



Data in Table 41 were used to plot the graphs in Figure 16.

Figure 16 shows that after the fourth program, the time to prepare these programs did not change substantially.

The "quality" of a linear program requires some definition. For the purpose of this thesis, quality might be defined as the degree of completeness with which the prepared program represented the actual farming business operation, and the degree of refinement of the data.

There is no doubt that the quality of the programs improved as the programmer gained experience. Due to the large mass of data, the programmer carried much information from one program to another. This resulted in a saving in time, but at the same time carried forward questionable assumptions from the previous programs. Re-examination of these assumptions occurred when substantial differences existed between the activities in the previous programs and the activities in the program being prepared. If Case 1 is compared to Case 11, it may be observed that much more detail is placed in each activity in the form of cost accounting in Case 11. As well, the basic representation improved.

The number of programs which must be prepared before complete representation of an actual farming business operation can be achieved is probably the most significant criticism of following the British example of linear programming. The method illustrated in American literature is much easier to learn and to use in preparing programs. However, once the procedure used in this project has been mastered by the programmer, the greater flexibility has much to offer.



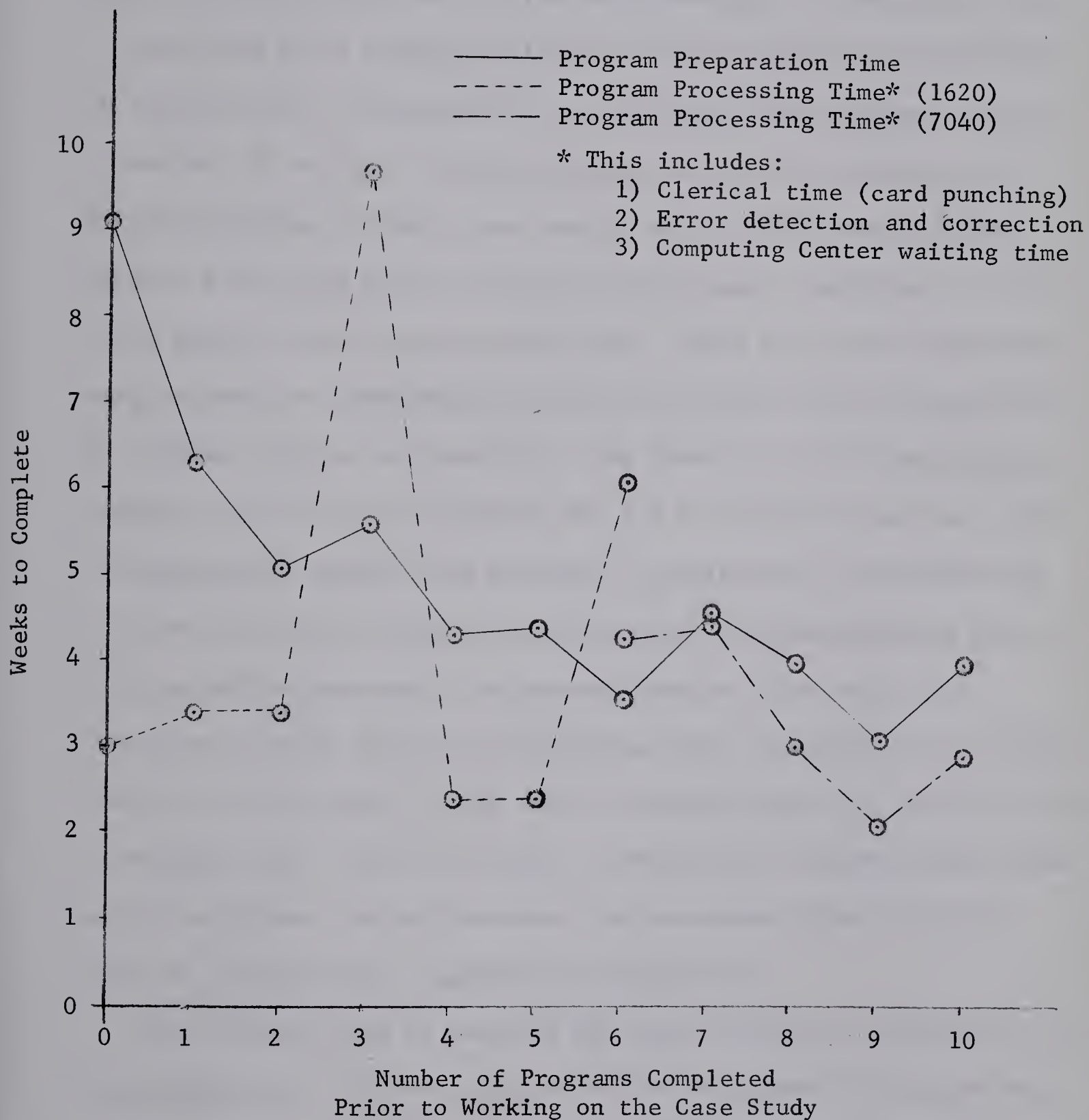


Figure 16. Programming Times for the Project.



The time required to process these problems represents a somewhat different story. It appears from Table 41 that the time taken and the number of trials taken bear little relationship. The reason for this is that there was a waiting period of one to two days on the IBM 1620 for the problems to be processed. On the 7040, this has been reduced to one-half to one day. As well, delays occurred in picking up the processed program. Thus it was hard to estimate the working time required before the final solution was obtained. The number of trials is the best indicator of processing time. Extra trials were generally required when the programmer did not take enough time in checking the data coding and the card punching. The number of trials per program averaged 3.5 on the 1620 computer and 3.6 on the 7040 computer. This is considerably greater than MacHardy<sup>35</sup> required for the programs in his thesis where he required one extra trial for approximately half of the programs he prepared. The required three to five trials with associated waiting time at the Computing Centre resulted in an elapsed time of about 3.0 weeks. Each trial required roughly two to four hours of checking time. Figure 16 shows a comparison of the processing time for all programs. As may be noted, the processing time fluctuated about an average value, regardless of experience.

The clerical duty of punching the data on cards is included in the processing time. The time required by the programmer to prepare the cards for the first trial of a program is summarized in Table 42.



Table 42. Card Punching and Card Checking Time for the First Trial.

Case Study	Time (Man Hours)		Programs Used	Matrix Size (Includes Slacks) (Row x Column)	Number of Cards Prepared	Entries Punched		Entries Checked	
	Punching Cards	Checking & Correcting				Per Man Hour	Per Man Hour	Per Man Hour	Per Man Hour
1	5.0	1.3	Wood <sup>52</sup> (Orig.) Krieger <sup>39A</sup> (re-run)	34 x 99	360	72	277		
2	4.0	.8	Wood <sup>52</sup> Krieger <sup>39A</sup> (re-run)	33 x 97	362	91	453		
3	4.0	.7	Wood <sup>53</sup>	36 x 99	457	114	653		
12	1.5	.5	Krieger <sup>39A</sup>	26 x 90	190	127	380		
4	4.8	.8	Nickel & Davis <sup>39B</sup>	38 x 95	440	92	550		
5	2.8	.5	Nickel & Davis <sup>39B</sup>	30 x 87	372	133	744		
6	2.5	.8	Nickel & Davis <sup>39B</sup>	36 x 106	566	226	708		
7	2.4	1.6	Nickel & Davis <sup>39B</sup>	39 x 98	463	193	289		
8	5.0	3.5	Clasen <sup>13</sup>	37 x 93	442	88	127		
9	3.0	1.0	Clasen <sup>13</sup>	40 x 98	414	138	414		
7	2.0	.8	Clasen <sup>13</sup>	39 x 98	463	232	581		
10	2.0	1.0	Clasen <sup>13</sup>	43 x 102	514	257	514		
11	3.7	2.0	Clasen <sup>13</sup>	43 x 115	520	141	260		

, 151 ,



A comparison of Table 42 and Table 41 shows that this time was not an appreciable proportion of the processing time, even when the card punch operators were not experienced. Figure 17 shows the increase in the speed of data preparation for the first trial. It should be mentioned that the first four programs were punched with one person operating the card punch and one person calling the numbers. For all subsequent case studies, the card preparation was done by the programmer. Also the input format was the same in both of Wood's programs<sup>52, 53</sup>, while Krieger<sup>39A</sup>, and Nickel and Davis<sup>39B</sup> had formats very similar to one another but very different from Wood's. Clasen's<sup>13</sup> input format was again very different from Nickel and Davis<sup>39B</sup>.

While card punching time would be substantially less if the operator were skilled in card punch operation and completely familiar with the input format, checking time per trial can be expected to remain near these values.

#### 4.2 COMPUTER TIME

The actual running time on the computer varies considerably. The time could not be obtained from the IBM 1620 as the Computing Centre could not keep an accurate record. The time was estimated to be two to four hours on the average program. The IBM 7040 on the other hand, required the times shown in Table 43.



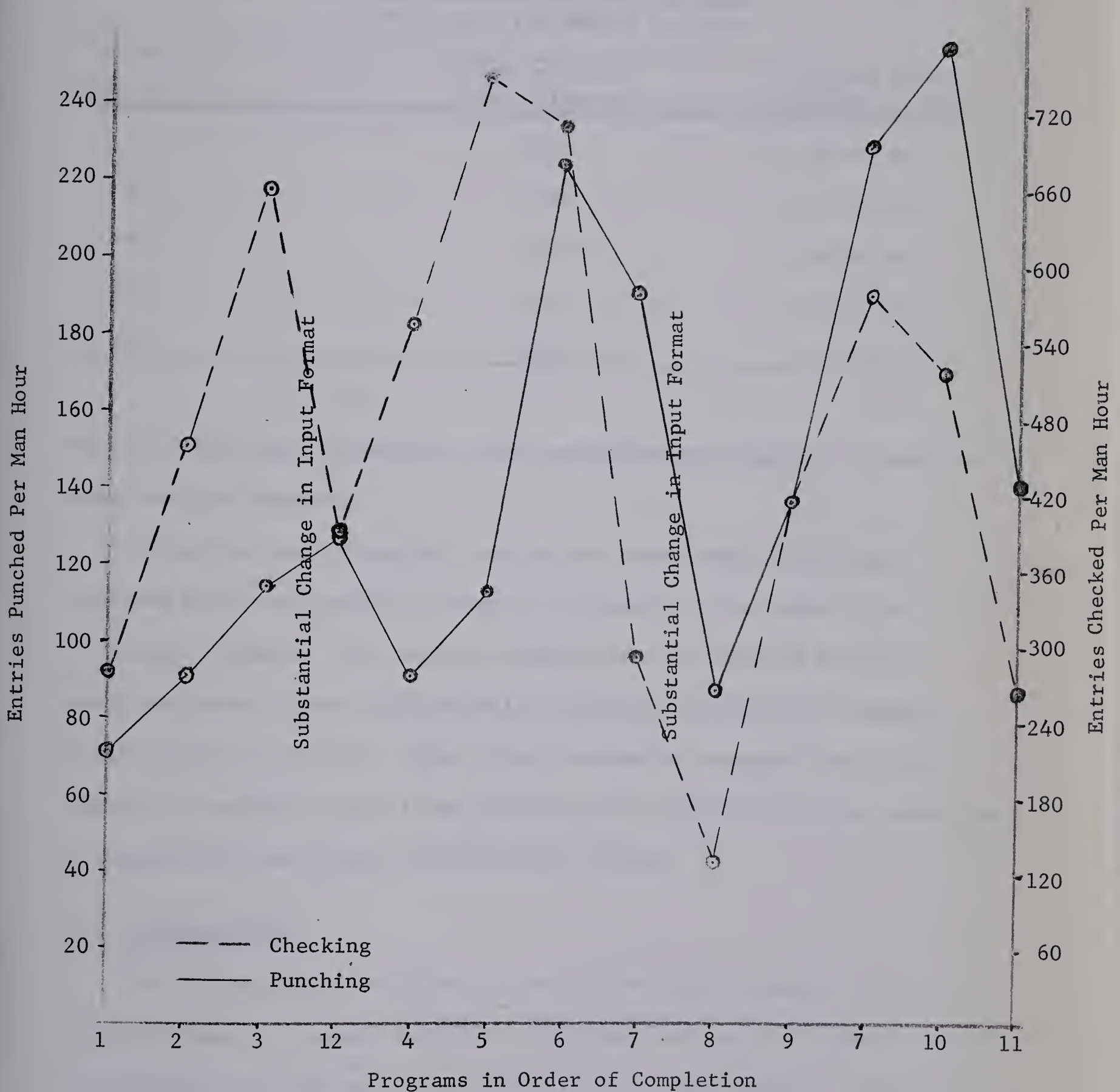


Figure 17. Data Preparation Time for First Trial.



Table 43. Time Taken on IBM 7040 to Solve  
the Linear Programming Problems.

Case Study	Time for Last Trial (Minutes)	Program Size (Including Slacks)
7	2.0	36 x 96
8	2.0	37 x 103
9	1.9	40 x 98
10	1.8	43 x 102
11	2.1	43 x 115

Of the times shown in Table 43, over one minute was required to read the data into the computer.

It may be noted then, that solving the farm linear programming problems with the computers presently available is no longer time consuming. However, the computer program for the IBM 1620 computer gave considerably more post optimal information than did the computer program for the IBM 7040. This extra information accounts for a small amount of computing time of the problem on the IBM 1620, so the comparison of computing times should recognize this factor.

#### 4.3 COMMUNICATION

Due to the fact that there are only a few people familiar with the technique at present, and that the present methods for linear programming are based on the availability of a high capacity, high speed digital computer, the work must be done from one of the two largest cities in Alberta. Thus contact with farmers in Alberta necessitates either rather lengthy trips for either the programmer or the farmer, or contact by letter. For the most satisfactory results, a programmer should visit



the farm to see the actual plant. In the case studies, the initial visit required about two weeks for the twelve farmers. However, this covered all of the province as can be seen by the map in Figure 1. In this project, the problem of communication was not so great in the initial contact as it was in obtaining some data which were not obtained on the first visit. The incomplete data were partly due to the programmer not being totally familiar with the requirements of linear programming when the data were collected. However, oversights will occur and the data may not be available to the farmer at that time. For the case studies in this project, any missing data were obtained from records available at the Alberta Department of Agriculture, the grain companies, livestock buyers and other sources rather than re-visit the farm.

All subsequent contact with the farmer was made by a letter to the District Agriculturist. This was usually to return the results. An effort was made to include all necessary information in these letters but there is still a great deal of information which would be useful to the farmer which comes out in a discussion. This additional information is mainly the effect of changes in the farm plan which differ from the optimal farm plan. In a letter, the District Agriculturist may put a different interpretation to the meaning of the farm plan than is actually the case. Also, only the programmer is able to tell the farmer, in most cases, what changes will take place in his farm returns by a change in one of his restrictions.

Another problem with letter contact is the fact that the programmer, being so familiar with the technique and terms used, has a tendency to leave out much of the detailed information about the program which may be



necessary to understand the results. Thus the farmer accepts the results at face value and does not check to see what changes are necessary to get closer to his actual costs or production averages. It had been hoped that more farmers would ask for re-runs with modifications in the various activities which the individual felt were closer to his actual experience. Unfortunately, the results were accepted in most cases without any communication returned to the programmer.

As well as the problem of not communicating all the possible information to the farmer, some misunderstandings do occur. This was most noticeable in Case 12 where an error in the letter made the results rather questionable. The explanation of this communication is given in Section 3.12.5, but the result was a substantial amount of dissatisfaction on the part of the farmer before it was settled. Once all errors were rectified and the results explained, the farmer was well satisfied. However, the misunderstanding would not have happened if the farmer were near enough to discuss the results with the programmer.

#### 4.4 DATA

Much criticism has been levelled at linear programming for its supposedly stringent data requirements. McFarquhar<sup>33</sup> quotes Simpson as stating in his paper in "The Farm Economist" (Vol. 9, No. 7, "Linear Programming for Increased Farm Profits") that linear programming will remain primarily a research tool because the data is too difficult to obtain to merit work on individual farm plans. However, Candler and Musgrave<sup>11</sup> and McFarquhar<sup>33</sup> point out that the linear programming technique uses the same accuracy of data as presently used techniques of budgeting



and marginal analysis. The difference is in the greater volume of data needed for a satisfactory linear program compared to that usually employed in budgeting or marginal analysis. McFarquhar<sup>33</sup> disagrees with Simpson and suggests linear programming will reach the farm planning scene because:

- a) it uses the same data as other techniques.
- b) it can handle a larger mass of data without the worker becoming lost.
- c) the computer does much of the tedious calculation.

If the accuracy of the coefficients of the case studies is to be criticized, it should be remembered that they should be criticized from the point of view of general accuracy rather than the view that linear programming has a special requirement for more accurate data than any other technique.

Many coefficients in the case studies differ in the number of significant digits carried. It was felt that rounding should occur in the solution rather than the matrix and that even if the last digits did not mean a great deal, entering them in the matrix was not time consuming. It was assumed that all figures were accurate to two figures, but any subsequent figures were open to question. This may raise a question about the value of calculating minor cost items such as oil and filter costs on tractors. It was felt that if the item was a cost, it should be considered, even if it changed only the third or fourth figure.

Reading over the data for these programs may produce some criticism on the basis that data for prices and yields other than the farmer's estimate should have been used. McFarquhar<sup>33</sup> disagrees with farmer



estimates and prefers libraries of average data. However, there are several advantages to using farmer estimates.

The primary advantage is that the responsibility of management is left with the farmer. Linear programming becomes a service which the farmer uses to help him sort out the vast volume of data he encounters in planning his farm. If he disagrees with the results, he only has his own data to blame and it makes him much more conscious of the necessity of having accurate data. Actual farm records of yields, grades, and prices for the period of farming are scarce, but it may be that some such technique as this may point out to the farmer the data which are required for the basis of his estimates.

A second advantage of using farmer estimates is that they allow for the differences in the skills of the farmer or the difference in the productivity of his land. Most average data available group the results of a number of farmers in the area. This does not allow for the variation in individual performance. Even average data for an individual farm may not give an accurate picture since they reflect past technology which may be inferior or superior to the technique presently in use.

The last advantage to be mentioned is that using farmer estimates where possible saves program preparation time. These estimates made up approximately one-half of the data required for each case study. The rest of the data was data from other sources. If farmer estimates are unsatisfactory, then all coefficients may be obtained using the data on average production and average prices which are available from several sources. The previous text gives a substantial number of these sources, but the list is far from exhaustive.



#### 4.5 SUBSEQUENT PROGRAMS

There were five operators who requested re-runs on their problems and supplied some changes in data. This is probably the area where much of the power of the technique lies. Preparation of the initial matrix would be an infrequent exercise for the farmer and would require a substantial amount of time. Most subsequent changes in the program take a very small amount of time as another restriction or activity may be added to the problem with only the fraction of work required for the initial program. More often than not, changes in the program are in the yields or returns estimates and the necessary program coefficients are altered by the amount of the change requested. On the five re-runs done for the farmers in the project, the changes were minor. A maximum of three hours were required to complete all the preparations of getting the data over to the Computing Centre, when the same computer program was used. Thus a farmer could have the program re-run every year by altering slightly the original program according to the new situation. The major cost would be in the first year when the matrix was prepared. In subsequent years costs would be considerably less. As well, when the card deck is saved, checking time is very much less than on the original run.

#### 4.6 SUGGESTIONS TO REDUCE THE WORK IN PREPARING LINEAR PROGRAMS

It is doubtful if preparation of linear programs in the manner that was used for this project is ever going to be very popular. There is a considerable number of repetitions of very simple procedures from activity to activity and program to program. If one individual were to spend all his time at preparing linear programs, he would soon



tend to lose interest, unless he were a very unique person. This is probably the reason that most farm linear programs prepared up to the present tend to be "benchmark" studies<sup>29, 30, 42, 48</sup>. As yet, little work has been done by any agency in Canada to offer this technique to Canadian farmers. Any agency contemplating offering widespread service is going to have to try to reduce the substantial amount of work in order to:

- a) spread the knowledge of the few personnel familiar with the technique and
- b) reduce the cost of giving the service.

The prime limitation is the number of people available to do applied linear programming. Presently, there are a few professional people capable of conducting linear programming studies because they have become interested in the technique for research purposes. However, it is doubtful if they would be interested in the application of the technique. The main source of personnel for practical linear programming seems to be the freshly trained graduate who has had some undergraduate training in linear programming procedure. There are a substantial number of recent University graduates who have an acquaintance with the procedure. However, the programmer in these case studies found that there was a substantial amount of material to master after being acquainted with the procedure before adequate linear programs could be prepared. This is illustrated in the quality of the programs from Case 1 to Case 11. This is mainly due to the fact that the basic principle of linear programming is easily and quickly taught. However, proficiency with linear programming comes only with an extensive amount of experience with the technique. It is the experienced individual who wants to continue in the daily use of the



technique that is in very limited supply.

The second limitation (cost) is not as severe, but is still a substantial obstacle.

This cost is a result of the large amount of time required per program. In this project, the time spent per program by an individual familiar with linear programming could be summarized as follows:

- a) One to one and one-half weeks to visit the farm, collect the data, and prepare the program.
- b) One week actual working time to process the problem and draft the reply to the farmer.

A total of approximately 2.5 weeks actual working time would be required per program. This agrees closely with Swanson<sup>47</sup> who suggested that two weeks work is required. McFarquhar<sup>33</sup> also suggests that the expense of preparing linear programs is the main obstacle due to the time per farm. However, he estimates that programming time per farm less than a day. This is very debatable in the light of this and other studies.

Using present procedures, commercial farm management consultants can expect only a few large farms to be interested. Also public extension agencies can hardly justify the cost of offering such a service to farmers. It is with these considerations in mind that the following suggestions were made.

#### 4.6.1 Use of the Decomposition Technique.

This technique, which was recently developed by Dantzig and Wolfe<sup>16</sup>, has much to offer in reducing the work required to prepare linear programming matrices in the manner that was used here. MacHardy<sup>34</sup> suggested the possibility of using the technique and Anderson<sup>3</sup> has enlarged the topic



to point out how farm linear programs, prepared as they have been for this project, fit the pattern of programs which may be decomposed. This is probably best illustrated by an example.

The matrix set up for Case 1 has been rearranged in Figure 18 into the order which shows the blocks of data clearly. Examination of the blocks shows that the first five equations pertain entirely to the cropping program, the next twelve equations pertain only to the livestock program, and the last sixteen plus the functional serve to link the two enterprises, as well as a few incidental activities. Therefore, it is obvious that the program may be solved using the decomposition technique.

There is a small point that should be mentioned at this point in connection with this technique. On looking at the linking equations, it will be noted that these equations constitute a large portion of the program. The question then arises as to the value of even trying to use the technique. With the advent of digital computers of the size and speed presently available, this is a valid question. However, for academic interest there is value in examining ways of reducing the size of the linking matrices. The only way to reduce the size is to reduce the number of equations. Examination of the equations shows that eight equations pertain to the available labor. These equations were placed in the original matrices for the following reasons:

- a) They offered an easy way to keep an account of the amount of labor required and when. Any extra time was hired as needed at a constant cost.
- b) Many programs from other studies<sup>8, 30, 42</sup> devoted a substantial part of the program to the labor.

Closer examination shows that labor on the Alberta scene may be regarded

[illegible]

Summer Feeds		Hay		Pellets (Maintenance)		Pellets (Production)		Supplement (Maintenance)		Supplement (Production)		Cow	Cow	Yearling Steer
Oats (Production)	Barley (Maintenance)	Barley (Production)	Hay (Maintenance)	100 lb	100 lb	100 lb	100 lb	100 lb	100 lb	100 lb	100 lb	49	50	53
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1</			

Figure 18. Linear Programming Matrix for Case 1 Rearranged to Show the Blocks of Coefficients.



more as an operating cost which is purchased as needed, much like fuel or oil. Few farmers keep a man full time if they only need him for six months of the year. Also there is a lot of casual labor that is hired only when the farmer is rushed. Since this is the present situation, the labor equations may be dropped and either replaced by one equation to enable a basic allowance of the farmer's time, or simply included as a direct operating cost per activity. In this case, the latter situation would be preferable since the object is to reduce the number of equations. In Case 1, this would mean that there would be only eight linking equations. The restricted master program would be reduced from 19 rows and 20 columns to 11 rows and 12 columns which is a considerable saving. Further adjustments in the program may even reduce the size further.

The major contribution of the decomposition principle is the proof it offers for the team approach to linear programming problems as suggested by Anderson<sup>3</sup> and MacHardy<sup>34</sup>. The problems with the programs in this project were the result of the programmer covering too many specialized fields. Discussion with the specialists in the various fields helped considerably, but much better programs could be prepared by the specialist if he knew the basic linear programming technique and could concern himself only with his specialty. The decomposition principle shows that specialists may prepare portions of the matrix and still maintain the continuity of the program. MacHardy<sup>34</sup> suggests going a step further and having "standard" blocks prepared for the different matrices. He suggests the total program be assembled by putting together the appropriate "standard" blocks which pertain to the area where the farm is located plus a small amount of calculation in the linking equations to include anything unique to the particular farm. Thus the actual work in programming individual farms is



small. Most of the work would be in the preparation of the "standard" blocks, and since they would be used several times, a substantial amount of care could go into the preparation of each standard block. The result would be good quality programs with a relatively small amount of programming time. However, up to the present time, these procedures have not been tested for practical programming problems and remain as suggestions.

The previous discussion has served to emphasize the value of the decomposition technique from the point of view of justifying the use of other procedures. Recognizing the fact that farm linear programming problems (prepared as they have been for this project) will decompose permits use of the actual procedure. At present, the procedure is fairly rigid as is illustrated in the appendix. Thus, solving the problem in this way does not hold very much appeal when compared to solving the whole problem on the available computers. However, more research will probably find ways of using the actual procedure of the decomposition technique.

#### 4.6.2 Standardized Form.

Another method to reduce the programming time required is to develop a standardized area program for different areas, with the coefficients that vary between farms, blank. This would be a linear program which listed the usual restrictions and activities of the area. This was not done in these case studies, as may be noted, if the figures showing the matrices are examined. The result was a matrix that did not show the blocks of data well, and tended to be somewhat difficult to organize when the program was being prepared. If a standard form were used, the farmer and programmer could then sit down and fill in the necessary data to complete a program relatively quickly. Most often the differences between



farms would be in the returns to an activity, the labor involved, the farming procedures and in the number and size of resources. Many of the ration balancing restrictions and crop selection restrictions would remain unchanged from farm to farm. The standard program should probably originally be set up by the team method suggested in Section 4.6.1. Any unusual activities or restrictions could then be added to the standard program with a minimum of effort. The standardization could even be carried a step further in listing the usual calculations to perform in order to obtain the desired matrix coefficients. This extension should probably be left at the point of example for the programmer to follow in order to keep the number of forms down to a minimum. As well, most of these calculations are basic to partial budgeting and should be familiar to anyone doing the work.

#### 4.7 IMPROVEMENTS AND EXTENSIONS

Examination of the matrices given in Section 3 will show that little work was done to fit the various linear programming improvements into the basic linear programming model. These include equipment selection developed by MacHardy<sup>34</sup>, flexibility and uncertainty developed by MacFarquhar<sup>32</sup>, seasonal supply of short term capital developed by Stewart<sup>45</sup>, extended planning horizons illustrated by Swanson<sup>47</sup>, integer programming developed by Gomory<sup>23</sup>, and many other techniques. The main reason for this was that the basic technique had to be mastered before branching out. It is only after these programs have been completed that the author feels familiar enough with the basic technique to extend the work gradually to include the newer developments. The disadvantage of including these techniques is that the problem size keeps increasing as more of the developments are



included. As the matrix size increases, it becomes more and more difficult for one person to handle. More often than not the programmer will avoid these developments to keep the program to manageable size, especially while beginning. Establishing standard procedures will help overcome this expansion in size. The computer programs will solve problems much larger than these illustrated here. Therefore the limit on size is primarily a psychological one at present.

The improvements discussed above are mainly in the line of mathematical alterations required to adjust linear mathematics in order to give a more complete representation of actual farm situations. As well as these program improvements, the smaller program adjustments made by the programmer in all his calculations as he gains experience are also important. The problems in this project could still be improved by using measures of adjusting standard costs such as machinery repair costs, veterinary costs, livestock losses to the values actually experienced by the individual operator. These minor improvements come only with the periodic review of the procedures and data used in calculating coefficients for the various activities.



## 5. CONCLUSIONS

This project was done to show some of the problems encountered in preparing linear programs for Alberta farmers. Presently, the most important obstacle is the problem of time spent by highly trained and very scarce personnel on each farmer's problem. As more and more details are included in the program, this becomes a bigger obstacle using present methods. However, there is a great deal of potential for streamlining the preparation of the matrix, which has not been examined from a practical point of view.

Communication with the farmer in Alberta and the tedious nature of the work are two problems which will also be encountered. However, if farmers show enough interest in the service, personal contact by means of the farmer doing much of the travelling, will solve this to a large extent. Also, standardized procedures will help to insure a very minimum of information is overlooked on the first visit. As far as the tedious nature of the work is concerned, it will be almost inversely proportional to the amount of refinement of the procedures.

Linear programming, however, still has much to offer the Alberta farmer for planning his farm. Farmers generally are being faced with many more alternative fields of production as time passes. This is especially true in the mixed farming areas of Alberta. Also, technology is changing at an increasing pace and this alters the comparative advantage of various crops and livestock enterprises. Previously, the farmer did most of the data sorting in his head. However, with the increase in volume of data, some technique is required. Since this procedure uses the same data as



budgeting techniques and has the advantages of

a) handling a larger volume of data at once,

b) using an electronic computer for some of the tedious calculation,

it is logical to expect linear programming to be the technique used. The problem is going to be the development of trained personnel, and the long term necessary to fully develop the potential of linear programming. No one has done this yet, as most of the linear programming work has been on benchmark and research studies. This is especially true in Canada.

It is easy to become disenchanted with the technique since the first programs are a great deal of work. Linear programming, therefore, will only become established in the near future if someone becomes interested in the practical application of the technique, has the ingenuity to develop better processing methods, and has the ability to gradually include the already developed improvements. It is probably going to reach the Alberta farmer as an accepted service eventually, but how close this eventuality is, depends upon those people who are at present, and those who will be in the future, working on linear programming.



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## 7. APPENDICES

### 7.1 THE PRINCIPLE OF DECOMPOSITION\*

The decomposition principle was developed by Dantzig and Wolfe<sup>16</sup> to reduce the size of linear programming problems. However, it is a technique that works only on special types of programs. The basis for the decomposition technique is the simplex algorithm using multipliers, although Hadley<sup>25</sup> develops the technique from the standard simplex algorithm for solving linear programs. The development here is based on that in Dantzig's text.<sup>14</sup>

#### 7.1.1 Basic Principle.

The decomposition technique will only apply to programs which have clearly defined sections or blocks in the program. The sections are a group of activities which have several restrictions pertaining only to that group and none other. For example, suppose we wish to solve the following linear programming problem,

$$\begin{array}{rcll} x_1 + x_2 & & \leq & 10 \\ & 2x_2 & \leq & 5 \\ & & x_3 + x_4 & \leq 20 \\ & & & x_4 \leq 5 \\ x_1 + x_2 + x_3 + x_4 & \leq & 20 \\ x_1 + \frac{1}{2}x_2 + 2x_3 + \frac{1}{4}x_4 & \leq & 20 \\ -2x_1 - 3x_2 - 4x_3 - x_4 & = & z(\min) \end{array}$$

\* This appendix is included to retain a delineation of the decomposition technique and an accompanying characteristic farm example.



By putting in the slack variables to change the equations to equalities and changing to matrix notation, the following matrix is obtained.

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$-z$	
1	1			1							= 10
	2				1						= 5
		1	1			1					= 20
			1				1				= 5
1	1	1	1					1			= 20
1	.5	2	.25						1		= 20
-2	-3	-4	-1							1	= 0

If the variables were re-arranged the matrix would be

$x_1$	$x_2$	$x_5$	$x_6$	$x_3$	$x_4$	$x_7$	$x_8$	$x_9$	$x_{10}$	$-z$	Constants
1	1	1									= 10
	2		1								= 5
				1	1	1					= 20
					1		1				= 5
1	1			1	1			1			= 20
1	.5			2	.25				1		= 20
-2	-3			-4	-1					1	= 0

To reduce notation let the matrix representation be as follows:

$A_1$  represents  $\begin{bmatrix} 1 & 1 & 1 & - \\ - & 2 & - & 1 \end{bmatrix}$

$A_2$  represents  $\begin{bmatrix} 1 & 1 & 1 & - \\ - & 1 & - & 1 \end{bmatrix}$



$$X_1 \text{ represents } \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_6 \end{bmatrix}$$

$$X_2 \text{ represents } \begin{bmatrix} x_3 \\ x_4 \\ x_7 \\ x_8 \end{bmatrix}$$

$$\bar{A}_1 \text{ represents } \begin{bmatrix} 1 & 1 & - & - \\ 1 & .5 & - & - \\ -2 & -3 & - & - \end{bmatrix}$$

$$\bar{A}_2 \text{ represents } \begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \end{bmatrix}$$

$$b_1 \text{ represents } \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$b_2 \text{ represents } \begin{bmatrix} 20 \\ 5 \end{bmatrix}$$

$$\bar{b} \text{ represents } \begin{bmatrix} 20 \\ 20 \\ 0 \end{bmatrix}$$

$$A_3 \text{ represents } \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X_3 \text{ represents } \begin{bmatrix} x_9 \\ x_{10} \end{bmatrix}$$

The sample problem could be represented as

$$A_1 X_1 = b_1$$

$$A_2 X_2 = b_2$$

$$\bar{A}_1 X_1 + \bar{A}_2 X_2 + A_3 X_3 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (-z) = \bar{b}$$

Let the column  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  be  $P_0$  which is standard notation for the column in many linear programming problems. Then the problem takes the form

$$A_1 X_1 = b_1$$

$$A_2 X_2 = b_2$$

$$\bar{A}_1 X_1 + \bar{A}_2 X_2 + A_3 X_3 - P_0 z = \bar{b}$$

Many of the farm linear programming problems take on this form. It would be convenient to solve the problem in smaller parts. The decomposition principle allows us to solve

$$\bar{A}_1 X_1 + \bar{A}_2 X_2 + A_3 X_3 - P_0 z = \bar{b}$$

subject to

$$A_1 X_1 = b_1$$

$$A_2 X_2 = b_2$$



For the present we will assume that there are bounded solutions to the restrictions  $A_1 X_1 = b_1$  and  $A_2 X_2 = b_2$ . These solutions will be extreme point solutions such that  $X_1$  has a number of different solutions.

$X_1 = X_{11}, X_{12}, X_{13}, \dots, X_{1k}$  and  $X_2 = X_{21}, X_{22}, X_{23}, \dots, X_{2m}$ . Any solution may be found by taking portions of the extreme point solutions and adding them together. For example, we can add one-half of the solution  $X_{11}$  and one-half of the solution  $X_{12}$  to get another solution to satisfy  $X_1$ . To state in mathematical terms

$$X_1 = \sum_{i=1}^k \Phi_i X_{1i} \quad \sum_{i=1}^k \Phi_i = 1 \quad \Phi_i \geq 0$$

where  $\Phi_i$  is the proportion of each extreme point  $X_{1k}$  used to make the solution desired. Similarly with  $X_2$ :

$$X_2 = \sum_{j=1}^m \mu_j X_{2j} \quad \sum_{j=1}^m \mu_j = 1 \quad \mu_j \geq 0$$

where  $\mu_j$  is the proportion of each extreme point solution of  $X_2$  used to give the desired solution. This means we wish to solve

$$\sum_{i=1}^k \Phi_i (\bar{A}_1 X_{1i}) + \sum_{j=1}^m \mu_j (\bar{A}_2 X_{2j}) + A_3 X_3 - P_0 z = \bar{b}$$

$$\sum_{i=1}^k \Phi_i = 1$$

$$\sum_{j=1}^m \mu_j = 1$$

$$X_3 \geq 0 \quad \Phi_i \geq 0 \quad \mu_j \geq 0$$

Using a linear transform,  $(\bar{A}_1 X_{1i})$  results in a column of numbers, given a particular solution of  $X_1$ . The same applies to  $(\bar{A}_2 X_{2j})$ . If



$$S_i = \bar{A}_1 X_{1i} \quad T_j = \bar{A}_2 X_{2j} \quad \text{the problem becomes}$$

$$\sum_{i=1}^k S_i \Phi_i + \sum_{j=1}^m T_j \mu_j + A_3 X_3 - P_0 z = \bar{b}$$

$$\sum_{i=1}^k \Phi_i = 1$$

$$\sum_{j=1}^m \mu_j = 1$$

$$X_3 \geq 0 \quad \Phi_i \geq 0 \quad \mu_j \geq 0$$

which is called the full master program.

The linear programming problem looks like

$\Phi_1$	$\Phi_2$	----	$\Phi_k$	$\mu_1$	$\mu_2$	----	$\mu_m$	$X_3$	$-z$		Constants
$S_1$	$S_2$	----	$S_k$	$T_1$	$T_2$	----	$T_m$	$A_3$	$P_0$	=	$\bar{b}$
1	1		1							=	1
				1	1		1			=	1

where we solve for  $\Phi_i$ ,  $\mu_j$ ,  $X_3$ , and  $-z$ . Fortunately not all the columns  $S_i$  and  $T_j$  are solved for before the program is started. Only those columns that enter the basic solution are generated as required. This would be a good point to examine the sample problem in the light of the above development. First the extreme point solutions for  $X_1$  and  $X_2$  will be generated.

$$A_1 X_1 = b_1 \quad \text{or} \quad \begin{bmatrix} 1 & 1 & 1 \\ & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$



$$X_{11} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 5 \end{bmatrix} \quad X_{12} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 5 \end{bmatrix} \quad X_{13} = \begin{bmatrix} 0 \\ 2.5 \\ 7.5 \\ 0 \end{bmatrix} \quad X_{14} = \begin{bmatrix} 7.5 \\ 2.5 \\ 0 \\ 0 \end{bmatrix}$$

$$A_2 X_2 = b_2 \text{ or } \begin{bmatrix} 1 & 1 & 1 & 1 \\ & 1 & & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} 20 \\ 5 \end{bmatrix}$$

$$X_{21} = \begin{bmatrix} 0 \\ 0 \\ 20 \\ 5 \end{bmatrix} \quad X_{22} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 5 \end{bmatrix} \quad X_{23} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix} \quad X_{24} = \begin{bmatrix} 15 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

Next the values of  $S_i$  and  $T_j$  are calculated.

$$S_i = \bar{A}_1 X_{1i} \quad T_j = \bar{A}_2 X_{2j}$$

$$S_1 = \begin{bmatrix} 1 & 1 & - & - \\ 1 & .5 & - & - \\ -2 & -3 & - & - \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} 1 & 1 & - & - \\ 1 & .5 & - & - \\ -2 & -3 & - & - \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -20 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 2.5 \\ 1.25 \\ -7.5 \end{bmatrix} \quad S_4 = \begin{bmatrix} 10 \\ 8.75 \\ -22.5 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 20 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad T_2 = \begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \end{bmatrix} \begin{bmatrix} 20 \\ 0 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ -80 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 5 \\ 1.25 \\ -5 \end{bmatrix} \quad T_4 = \begin{bmatrix} 20 \\ 31.25 \\ -65 \end{bmatrix}$$



Thus the sample problem would look like

$\Phi_1$	$\Phi_2$	$\Phi_3$	$\Phi_4$	$\mu_1$	$\mu_2$	$\mu_3$	$\mu_4$	$x_9$	$x_{10}$	$-z$		Constants
0	10	10	10	0	20	5	20	1	0	0	=	20
0	10	6.25	8.75	0	40	1.25	31.25	0	1	0	=	20
0	-20	-27.5	-22.5	0	-80	-5	-65	0	0	1	=	0
1	1	1	1	0	0	0	0	0	0	0	=	1
0	0	0	0	1	1	1	1	0	0	0	=	1

By solving for all the  $S_i$  and  $T_j$  we could use the simplex technique using multipliers on the whole program. This would be of no advantage. However, columns of  $S_i$  and  $T_j$  need be calculated only as they are required. To start solving the problem, one arbitrary column each of  $S_i$  and  $T_j$  are calculated and added to the matrix of  $A_3$ . For this example  $S_1$  and  $T_1$  have been selected. The program would appear as illustrated below and is known as the restricted master program.

Basis Variables		$\Phi_1$	$\mu_1$	$x_9$	$x_{10}$	$-z$	Constants
$x_9$		$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	1	0	0	20
$x_{10}$	$S_1$	0	0	$T_1$ 0	1	0	20
$-z$		$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	0	0	1	0
$\Phi_1$		1	0	0	0	0	1
$\mu_1$		0	1	0	0	0	1



When solving by hand, it is obvious that this is a feasible solution and that it is possible to proceed directly to Phase II of the simplex procedure. Later, a method for Phase I will be given where a feasible solution is not readily available. For the present the Phase II procedure will be used.

The first feasible solution is  $\Phi_1 = 1$ ,  $\mu_1 = 1$ ,  $x_9 = 20$ ,  $x_{10} = 20$ ,  $-z = 0$ . The next step is to determine whether or not this is the optimal solution. If not, the  $S_i$  or  $T_j$  which enter the basis must be selected and entered. Then the procedure is repeated until an optimal solution is reached. The detailed procedure follows.

First, the value of the multipliers must be determined. Since the first matrix is in canonical form, it can be arranged in a diagonal matrix if the  $\Phi_1$  and  $\mu_1$  columns were placed after the  $x_9$ ,  $x_{10}$ , and  $-z$  columns as is shown below.

Basis Variable	$x_9$	$x_{10}$	$-z$	$\Phi_1$	$\mu_1$	Constants
$x_9$	1	0	0	0	0	20
$x_{10}$	0	1	0	0	0	20
$-z$	0	0	1	0	0	0
$\Phi_1$	0	0	0	1	0	1
$\mu_1$	0	0	0	0	1	1

The above matrix is also the inverse of the starting basis and the multipliers are the values entered in the  $-z$  row. In this problem, they are  $[0 \ 0 \ 1 \ 0 \ 0]$ . The multipliers associated with all rows previous to the



$\Phi_i$  and  $\mu_j$  rows are usually represented symbolically by  $\pi$ . The multiplier associated with the  $\Phi_i$  row may be represented as  $-s$  and the multiplier associated with the  $\mu_j$  row represented as  $-t$ . In the sample problem  $\pi^0$  is  $[0 \ 0 \ 1]$ ,  $-s^0$  is 0, and  $-t^0$  is 0. When a solution is at hand the next step is to determine whether or not the optimum has been reached. The multipliers have the relationship that  $\pi P_0 = 1$ ,  $\pi S_i = s$ ,  $\pi T_j = t$ , where  $\pi$ ,  $s$ ,  $t$ ,  $S_i$ , and  $T_j$  are known from the present solution. Presently the problem has

$$\pi^0 = [0 \ 0 \ 1]$$

Therefore

$$\pi^0 S_1 = [0 \ 0 \ 1] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 = s^0$$

and

$$\pi^0 T_1 = [0 \ 0 \ 1] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 = t^0$$

Now the column with the lowest value to the formulae  $\pi^0 S_i - s^0 < 0$  or  $\pi^0 T_j - t^0 < 0$  is the column which will enter the basis. Therefore the minimum value for  $\pi^0 S_i$  and the minimum value for  $\pi^0 T_j$  must be calculated. This is the same as stating that the solutions to

$$\text{Min. } (\pi^0 \overline{A}_1) X_1$$

$$\text{and Min. } (\pi^0 \overline{A}_2) X_2 \text{ are required.}$$

The solution for  $X_1$  and  $X_2$  can be obtained by solving the linear program  $L_1$

$$A_1 X_1 = b_1$$

$$X_1 \geq 0$$

$$(\pi^0 \overline{A}_1) X_1 = z_1 \text{ (min.)}$$

and the linear program  $L_2$

$$A_2 X_2 = b_2$$

$$X_2 \geq 0$$

$$(\pi^0 \overline{A}_2) X_2 = z_2 \text{ (min.)}$$



For the sample problem, the programs would be developed as illustrated below.

$$\pi^0 = [0 \ 0 \ 1] \quad \bar{A}_1 = \begin{bmatrix} 1 & 1 & - & - \\ 1 & .5 & - & - \\ -2 & -3 & - & - \end{bmatrix}$$

$$\text{Min. } (\pi^0 \bar{A}_1) X_1 = \text{Min. } [-2 \ -3 \ 0 \ 0] X_1$$

The sub-program  $L_1$  becomes

$x_1$	$x_2$	$x_5$	$x_6$		Constants
1	1	1		=	10
	2		1	=	5
-2	-3			=	$z_1^1$

If a column is added for  $-z_1^1$ , the problem appears in matrix form as

$x_1$	$x_2$	$x_5$	$x_6$	$-z_1^1$		Constants
1	1	1			=	10
	2		1		=	5
-2	-3			1	=	0

By usual linear programming procedure, it would be normal to start with a Phase I procedure to place some matrix columns in canonical form. However, this procedure may be deleted if there is already a starting basis where each row has one column in canonical form. Such is the case in the sub-program  $L_1$ , and therefore the procedure for Phase II is initiated directly. The basis variables are  $x_5$ ,  $x_6$ ,  $-z_1^1$ , and these have



an inverse which is a diagonal matrix with unity entries in the diagonal and zero elsewhere. The multipliers technique is then used to solve the problem.

Basis Variable	$x_5$	$x_6$	$-z_1$	Constants	Entering Variable
$x_5$	1			10	$x_2$ 1
$x_6$		1		5	<u>2</u> Pivot
$-z_1$			1	0	-3

Multipliers:  $[0 \ 0 \ 1]$

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j =$	-2	-3	0	0

Basis Variable	$x_5$	$x_6$	$-z_1$	Constants	Entering Variable
$x_5$	1	-.5		7.5	$x_1$ <u>1</u> Pivot
$x_2$		.5		2.5	0
$-z_1$		1.5	1	7.5	-2

Multipliers:  $[0 \ 1.5 \ 1]$

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j =$	-2	0	0	1.5



Basis Variable	$x_5$	$x_6$	$-z_1^1$	Constants
$x_1$	1	-.5	0	7.5
$x_2$	0	.5	0	2.5
$-z$	2	.5	1	22.5

Multipliers =  $[2 \ .5 \ 1]$

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j$	0	0	2	.5

Solution to  $L_1$ :

$$X_{12} = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 2.5 \\ 0 \\ 0 \end{bmatrix}; \quad -z_1^1 = 22.5 \quad \text{or} \quad z_1^1 = -22.5$$

$$\pi^0 s_2 = (\pi^0 \bar{A}_1) X_{12} = z_1^1 = -22.5$$

$$\pi^0 s_2 - s^0 = -22.5 - 0 = -22.5 < 0$$

Similarly for the sub-program  $L_2$

$$\text{Min.}(\pi^0 \bar{A}_2) X_2 = [0 \ 0 \ 1] \begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \end{bmatrix} X_2 = \text{Min.} [-4 \ -1 \ 0 \ 0] X_2$$

This row is the minimizing row added to  $A_2 X_2 = b_2$  so the program appears as

$x_3$	$x_4$	$x_7$	$x_8$		Constants
1	1	1		=	20
	1		1	=	5
-4	-1			=	$z_2^1$



Adding the  $-z_2^1$  column

$x_3$	$x_4$	$x_7$	$x_8$	$-z_2^1$		Constants
1	1	1			=	20
	1		1		=	5
-4	-1			1	=	0

Again, the matrix has two columns in canonical form so Phase II of the simplex procedure using multipliers is applied. The sub-program  $L_2$  is solved as follows:

Basis Variable	$x_7$	$x_8$	$-z_2^1$	Constants	Entering Variable
$x_7$	1			20	$x_3$ <span style="border: 1px solid black;">1</span> Pivot
$x_8$		1		5	0
$-z_2^1$			1	0	-4

Multipliers =  $[0 \ 0 \ 1]$

	$x_3$	$x_4$	$x_7$	$x_8$
$\bar{c}_j$	= -4	-1	0	0



Basis Variable	$x_7$	$x_8$	$-z_2^1$	Constants
$x_3$	1	0	0	20
$x_8$	0	1	0	5
$-z_2^1$	4	0	1	80

Multipliers =  $[4 \ 0 \ 1]$

	$x_3$	$x_4$	$x_7$	$x_8$
$\bar{c}_j$	= 0	3	4	0

Solution to  $L_2$ :

$$X_{22} = \begin{bmatrix} x_3 \\ x_4 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 5 \end{bmatrix}$$

and

$$\pi^0_{T_2} = (\pi^0_{\bar{A}_2})X_2 = z_2^1 = -80$$

so

$$\pi^0_{T_2} - t^0 = -80 - 0 = -80$$

The column entering the restricted master program is the column which has the most negative of  $\pi^0_{S_i} - s^0$  or  $\pi^0_{T_j} - t^0$  which in this case is  $T_2$ . This is prepared for entry into the restricted master program.

$$T_2 = \bar{A}_2 X_{22} = \begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \end{bmatrix} \begin{bmatrix} 20 \\ 0 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ -80 \end{bmatrix}$$

When the entries are added to indicate the correct multipliers for the column,

the column becomes  $\begin{bmatrix} 20 \\ 40 \\ -80 \\ 0 \\ 1 \end{bmatrix}$



From here the standard procedure of the simplex technique using multipliers applies. Thus the column must be adjusted according to the inverse.

Therefore the entries of the entering variable column are:

$$\text{First: } \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 20 \\ 40 \\ -80 \\ 0 \\ 1 \end{bmatrix} = 20 \quad \text{Second: } \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 20 \\ 40 \\ -80 \\ 0 \\ 1 \end{bmatrix} = 40$$

$$\text{Third: } \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 20 \\ 40 \\ -80 \\ 0 \\ 1 \end{bmatrix} = -80 \quad \text{Fourth: } 0 \quad \text{Fifth: } 1$$

In entering this variable no pivot is allowed on the -z row. The restricted master program is solved as follows:

Basis Variable	$x_9$	$x_{10}$	-z	$\Phi_1$	$\mu_1$	Constants	Entering Variable
							$\mu_2$
$x_9$	1	0	0	0	0	20	20
$x_{10}$	0	1	0	0	0	20	<u>40</u> Pivot
-z	0	0	1	0	0	0	-80
$\Phi_1$	0	0	0	1	0	1	0
$\mu_1$	0	0	0	0	1	1	1



Basis Variable	$x_9$	$x_{10}$	$-z$	$\Phi_1$	$\mu_1$	Constants
$x_9$	1	-.5	0	0	0	10
$\mu_2$	0	.025	0	0	0	.5
$-z$	0	2	1	0	0	40
$\Phi_1$	0	0	0	1	0	1
$\mu_1$	0	-.025	0	0	1	.5

The problem is now in the same state as it was at the beginning of the explanation. The procedure of checking to see if this solution is optimal and of entering a new column into the restricted master program when the solution is not optimal is repeated until the optimum solution is reached. The example will be carried to solution without further explanation.

$$\pi^1 = [0 \ 2 \ 1]$$

$$\pi^1 P_0 = 1 = [0 \ 2 \ 1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1$$

$$\pi^1 S_1 = [0 \ 2 \ 1] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 = s^1$$

$$\pi^1 T_1 = [0 \ 2 \ 1] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 = t^1$$

$$\pi^1 T_2 = [0 \ 2 \ 1] \begin{bmatrix} 20 \\ 40 \\ -80 \end{bmatrix} = 0 = t^1$$



Solving  $L_1$ :

$$\pi^1 \bar{A}_1 = [0 \ -2 \ 0 \ 0]$$

$$\begin{bmatrix} 1 & 1 & 1 & - \\ - & 2 & - & 1 \\ 0 & -2 & 0 & 0 \end{bmatrix} X_{13} = \begin{bmatrix} 10 \\ 5 \\ z_1 \end{bmatrix}$$

Matrix notation

$x_1$	$x_2$	$x_5$	$x_6$	$-z_1^2$	Constants
1	1	1			10
	2		1		5
	-2			1	0

Using simplex multipliers and Phase II

Basis Variable	$x_5$	$x_6$	$-z_1^2$	Constants	Entering Variable
$x_5$	1			10	$x_2$
$x_6$		1		5	<u>2</u>
$-z_1^2$			1	0	-2

$$\text{Multipliers} = [0 \ 0 \ 1]$$

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j$	= 0	-2	0	0



Basis Variable	$x_5$	$x_6$	$-z_1^2$	Constants
$x_5$	1	-.5		7.5
$x_2$		.5		2.5
$-z_1^2$		1	1	5

Multipliers =  $[0 \ 1 \ 1]$

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j$	= 0	0	0	1

$$X_{13} = \begin{bmatrix} 0 \\ 2.5 \\ 7.5 \\ 0 \end{bmatrix}$$

$$\pi^1 \bar{A}_1 X_{13} = z_1^2 = -5$$

$$\pi^1 s_3 - s^1 = -5 - 0 = -5$$

Solving  $L_2$ :

$$\pi^1 \bar{A}_2 = [0 \ -0.5 \ 0 \ 0]$$

$$\begin{bmatrix} 1 & 1 & 1 & - \\ - & 1 & - & 1 \\ 0 & -.5 & 0 & 0 \end{bmatrix} X_{23} = \begin{bmatrix} 20 \\ 5 \\ z_2^2 \end{bmatrix}$$

Matrix notation

$x_3$	$x_4$	$x_7$	$x_8$	$-z_2^2$	Constants
1	1	1			20
	1		1		5
	-.5			1	0



Using simplex multipliers and Phase II

Basis Variable	$x_7$	$x_8$	$-z_2^2$	Constants	Entering Variable
$x_7$	1			20	$x_4$ 1
$x_8$		1		5	$\boxed{1}$
$-z_2^2$			1	0	-.5

$$\text{Multipliers} = [\bar{0} \ 0 \ 1]$$

	$x_3$	$x_4$	$x_7$	$x_8$
$\bar{c}_j$	= 0	-.5	0	0

Basis Variable	$x_7$	$x_8$	$-z_2^2$	Constants
$x_7$	1	-1		15
$x_4$		1		5
$-z_2^2$		.5	1	2.5

$$\text{Multipliers} = [\bar{0} \ .5 \ 1]$$

	$x_3$	$x_4$	$x_7$	$x_8$
$\bar{c}_j$	= 0	0	0	.5

$$X_{23} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix}$$

$$\pi_{A_2}^1 X_{23} = z_2^2 = -2.5$$

$$\pi_{T_3}^1 - t^1 = -2.5 - 0 = -2.5$$

The entering column is  $S_3$  or  $\begin{bmatrix} 1 & 1 & - & - \\ 1 & .5 & - & - \\ -2 & -3 & - & - \end{bmatrix} \begin{bmatrix} 0 \\ 2.5 \\ 7.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1.25 \\ -7.5 \end{bmatrix}$



Since this has a  $\Phi$  multiplier, the entry for the  $\Phi$  row in the restricted master program is 1 and the entry for the  $\mu$  row is 0. The entering

column is  $\begin{bmatrix} 2.5 \\ 1.25 \\ -7.5 \\ 1 \\ 0 \end{bmatrix}$  and enters the restricted master program after being modified to the inverse.

Basis Variable	$x_9$	$x_{10}$	$-z$	$\Phi_1$	$\mu_1$	Constants	Entering Variable
$x_9$	1	-.5	0	0	0	10	$\Phi_3$ 1.875
$\mu_2$	0	.025	0	0	0	.5	.031
$-z$	0	2	1	0	0	40	-5
$\Phi_1$	0	0	0	1	0	1	<span style="border: 1px solid black;">1</span>
$\mu_1$	0	-.025	0	0	1	.5	-.031

Basis Variable	$x_9$	$x_{10}$	$-z$	$\Phi_1$	$\mu_1$	Constants
$x_9$	1	-.5	0	-1.875	0	8.125
$\mu_2$	0	.025	0	-.031	0	.469
$-z$	0	2	1	5	0	45
$\Phi_3$	0	0	0	1	0	1
$\mu_1$	0	-.025	0	.031	1	.531



$$\pi^2 = [0 \ 2 \ 1]$$

$$\pi^2 P_0 = [0 \ 2 \ 1] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1$$

$$\pi^2 S_3 = [0 \ 2 \ 1] \begin{bmatrix} 2.5 \\ 1.25 \\ -7.5 \end{bmatrix} = -5 = s^2$$

$$\pi^2 T_1 = [0 \ 2 \ 1] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 = t^2$$

$$\pi^2 T_2 = [0 \ 2 \ 1] \begin{bmatrix} 20 \\ 40 \\ -80 \end{bmatrix} = 0 = t^2$$

The multipliers  $\pi^2$  are identical to  $\pi^1$  so the results of the sub-program are identical to the previous solutions. Therefore  $X_{13} = X_{14}$  and  $X_{23} = X_{24}$

$$\text{or } X_{14} = \begin{bmatrix} 0 \\ 2.5 \\ 7.5 \\ 0 \end{bmatrix} \quad \text{and} \quad X_{24} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix}$$

$$\text{Therefore } (\pi^2 A_1) X_{14} = z_1^3 = z_1^2 = -5 \quad \text{and}$$

$$(\pi^2 A_2) X_{24} = z_2^3 = z_2^2 = -2.5$$

$$\pi^2 S_4 - s^2 = -5 - (-5) = 0$$

$$\pi^2 T_4 - t^2 = -2.5 - 0 = -2.5$$

Therefore the next column entering is  $T_4$  which is

$$\begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 1.25 \\ -5 \end{bmatrix}$$

Since this has a  $\mu_j$  multiplier associated, the column becomes

$$\begin{bmatrix} 5 \\ 1.25 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$



and the program is solved as follows:

Basis Variable	$x_9$	$x_{10}$	-z	$\Phi_1$	$\mu_1$	Constants	Entering Variable
							$\mu_4$
$x_9$	1	-.500	0	-1.875	0	8.125	3.375
$\mu_2$	0	.025	0	-.031	0	.469	.031
-z	0	2.000	1	5	0	45	-2.5
$\Phi_3$	0	0	0	1	0	1	0
$\mu_1$	0	-.025	0	.031	1	.531	<span style="border: 1px solid black;">.969</span>

Basis Variable	$x_9$	$x_{10}$	-z	$\Phi_1$	$\mu_1$	Constants
$x_9$	1	-.398	0	-2.016	-4.516	5.72
$\mu_2$	0	.026	0	-.032	-.032	.452
-z	0	1.935	1	5.081	2.580	46.371
$\Phi_3$	0	0	0	1	0	1
$\mu_4$	0	-.026	0	.032	1.032	.548

Multipliers:  $\pi^3 = [0 \ 1.935 \ 1]$      $s^3 = -5.081$      $t^3 = -2.580$

The functional row to sub-program  $L_1$  is:

$$[-.064 \ -2.032 \ 0 \ 0]$$



Solving  $L_1$ :

Program

$x_1$	$x_2$	$x_5$	$x_6$	$-z_1^4$	Constants
1	1	1			10
	2		1		5
-.064	-2.032			1	0

Solution

Basis Variable	$x_5$	$x_6$	$-z_1^4$	Constants	Entering Variable
$x_5$	1			10	$x_2$ 1
$x_6$		1		5	[2]
$-z_1^4$			1	0	-2.032

Multipliers: [0 0 1]

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j$	-.064	-2.032	0	0

Basis Variable	$x_5$	$x_6$	$-z_1^4$	Constants	Entering Variable
$x_5$	1	-.5		7.5	$x_1$ [1]
$x_2$		.5		2.5	0
$-z_1^4$		1.016	1	5.081	-.064

Multipliers: [0 1.016 1]

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j$	-.064	0	0	1.016



Basis Variable	$x_5$	$x_6$	$-z_1^4$	Constants
$x_1$	1	-.5		7.5
$x_2$		.5		2.5
$-z_1^4$	.064	0.984	1	5.565

Multipliers:  $[\text{.064 } 0.984 \text{ } 1]$

	$x_1$	$x_2$	$x_5$	$x_6$
$\bar{c}_j$	= 0	0	.064	0.984

Therefore

$$z_1^4 = -5.565$$

$$z_1^4 - s^3 = -5.565 - (-5.081) = -.484$$

The functional row to the sub-program  $L_2$  is

$$[-.121 \text{ } -.594 \text{ } 0 \text{ } 0]$$

Solving  $L_2$ :

Program

$x_3$	$x_4$	$x_7$	$x_8$	$-z_2^4$	Constants
1	1	1			20
	1		1		5
-.129	-.594			1	0



Solution

Basis Variable	$x_7$	$x_8$	$-z_2^4$	Constants	Entering Variable
$x_7$	1			20	$x_4$ 1
$x_8$		1		5	$\boxed{1}$
$-z_2^4$			1	0	-.594

Multipliers:  $[0 \ 0 \ 1]$

	$x_3$	$x_4$	$x_7$	$x_8$
$\bar{c}_j$	= -.129	-.594	0	0

Basis Variable	$x_7$	$x_8$	$-z_2^4$	Constants	Entering Variable
$x_7$	1	-1		15	$x_3$ $\boxed{1}$
$x_4$		1		5	0
$-z_2^4$		.594	1	2.580	-.129

Multipliers:  $[0 \ .594 \ 1]$

	$x_3$	$x_4$	$x_7$	$x_8$
$\bar{c}_j$	= -.129	0	0	.594

Basis Variable	$x_7$	$x_8$	$-z_2^4$	Constants
$x_3$	1	-1		15
$x_4$		1		5
$-z_2^4$	.258	.258	1	4.516

Multipliers:  $[.258 \ .258 \ 1]$

	$x_3$	$x_4$	$x_7$	$x_8$
$\bar{c}_j$	= 0	0	.258	.258



Therefore

$$z_2^4 = -4.516 \quad \text{and} \quad z_2^4 - t^3 = -4.516 - (-2.580) \\ = -1.936$$

The solution used is  $X_{25} = \begin{bmatrix} 15 \\ 5 \\ 0 \\ 0 \end{bmatrix}$  Then  $\bar{A}_2 X_{25} = T_5 = \begin{bmatrix} 20 \\ 31.25 \\ -65 \end{bmatrix}$

By solving the restricted master program using  $\bar{A}_2 X_{25} = T_5$  which is associated with the variable  $\mu_5$ , the results are as follows:

Basis Variable	$x_9$	$x_{10}$	-z	$\Phi_1$	$\mu_1$	Constants	Entering Variable
							$\mu_5$
$x_9$	1	-.398	0	-2.016	-4.516	5.72	3.387
$\mu_2$	0	.026	0	-.032	-.032	.452	<span style="border: 1px solid black;">.774</span>
-z	0	1.935	1	5.081	2.580	46.371	-1.936
$\Phi_3$	0	0	0	1	0	1	0
$\mu_4$	0	-.026	0	.032	1.032	.548	.226

Basis Variable	$x_9$	$x_{10}$	-z	$\Phi_1$	$\mu_1$	Constants
$x_9$	1	-.5	0	1.875	-4.365	3.75
$\mu_5$	0	.033	0	-.042	-.042	.583
-z	0	2.0	1	5	2.5	47.5
$\Phi_3$	0	0	0	1	0	1
$\mu_4$	0	-.033	0	-.042	1.042	.417

$$\pi^4 = [0 \ 2 \ 1]$$

$$-s^4 = 5$$

$$-t^4 = 2.5$$



The solutions  $X_{16}$  and  $X_{26}$  are identical to the solutions  $X_{13}$  and  $X_{23}$ , as  $\pi^4$  is the same as  $\pi^1$ . Therefore  $z_1^5 = z_1^2 = -5$  and  $z_2^5 = z_2^2 = -2.5$ . Then  $\pi^4 S_6 - s^4 = -5 - (-5) = 0$  and  $\pi^4 T_6 - t^4 = -2.5 - (-2.5) = 0$ .

This indicates that the optimum solution has been reached. This solution reads in proportions of solutions to the sub-programs. In this case,

$x_9 = 3.75$ ,  $\mu_5 = .583$ ,  $z = -47.5$ ,  $\Phi_3 = 1$ ,  $\mu_4 = .417$ . Therefore the functional has the value of  $-47.5$ ,  $x_9 = 3.75$ ,  $.583$  of solution  $X_{25}$  is used,  $.417$  of solution  $X_{24}$  is used, and all of solution  $X_{13}$  is used.

To find the solution in terms of activities,

$$X_{25} = \begin{bmatrix} x_3 \\ x_4 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 15 \\ 5 \\ 0 \\ 0 \end{bmatrix} .583 = \begin{bmatrix} 8.75 \\ 2.918 \\ 0 \\ 0 \end{bmatrix}$$

$$X_{24} = \begin{bmatrix} x_3 \\ x_4 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix} .417 = \begin{bmatrix} 0 \\ 2.083 \\ 6.25 \\ 0 \end{bmatrix}$$

$$\begin{aligned} X_2 &= X_{25}(.583) + X_{24}(.417) \\ &= \begin{bmatrix} 8.75 \\ 2.918 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2.083 \\ 6.25 \\ 0 \end{bmatrix} = \begin{bmatrix} 8.75 \\ 5 \\ 6.25 \\ 0 \end{bmatrix} \end{aligned}$$

Therefore  $x_3 = 8.75$ ,  $x_4 = 5$ ,  $x_7 = 6.25$ ,  $x_8 = 0$

$$X_1 = X_{13} = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 2.5 \\ 7.5 \\ 0 \end{bmatrix}$$

Therefore  $x_1 = 0$ ,  $x_2 = 2.5$ ,  $x_5 = 7.5$ ,  $x_6 = 0$

The final solution then is  $x_2 = 2.5$ ,  $x_3 = 8.75$ ,  $x_4 = 5.0$ ,  $x_5 = 7.5$ ,  $x_7 = 6.25$ ,  $x_9 = 3.75$ ,  $x_1 = x_6 = x_8 = x_{10} = 0$ .



### 7.1.2 Homogeneous Solutions.

A special case often encountered in farm linear programs is the case of homogeneous solutions. This occurs when

$$A_1 X_{1i} = 0 \quad \text{or} \quad A_2 X_{2j} = 0$$

$$(\pi \bar{A}_1) X_{1i} \leq 0 \quad \text{or} \quad (\pi \bar{A}_2) X_{2j} \leq 0$$

A programming problem may result in a number of homogeneous and a number of non-homogeneous solutions. If this is the case, non-negative combinations of homogeneous solutions as well as the non-homogeneous solutions are of interest. This is done by changing the equations of the master program to the following:

$$\sum_{i=1}^k S_i \bar{\Phi}_i + \sum_{j=1}^m T_j \bar{\mu}_j + A_3 X_3 - P_0 z = \bar{b}$$

$$\sum_{i=1}^k \delta_i \bar{\Phi}_i = 1$$

$$\sum_{j=1}^m \delta_j \bar{\mu}_j = 1$$

where  $\delta_i = 0$  if  $X_{1i}$  is a homogeneous solution  
 $\delta_i = 1$  if  $X_{1i}$  is a basic feasible solution  
 $\delta_j = 0$  if  $X_{2j}$  is a homogeneous solution  
 $\delta_j = 1$  if  $X_{2j}$  is a basic feasible solution

Now if  $b_1 = 0$  or  $b_2 = 0$ , only homogeneous solutions are of interest. Thus if  $b_1 = 0$ , all  $\delta_i = 0$  and the restriction  $\sum_{i=1}^k \delta_i \bar{\Phi}_i = 1$  may be dropped since it becomes meaningless. The same may be done to  $\bar{\mu}_j$  if  $b_2 = 0$ . This is often the case with farm linear programs and thus it presents the problem of solving a matrix for the optimum of a homogeneous system. Normal linear programming procedure only gives a trivial solution of all  $x$ 's being



equal to zero. This is meaningless so some way must be found to solve the system to get a non-trivial solution. Dantzig<sup>14</sup> mentions several theorems (pages 136-138) given by early mathematicians which may be used to solve the systems. The one used here is the one Dantzig<sup>14</sup> credits to Gordon. This theorem states that if  $\sum_{j=1}^n a_{ij}x_j = 0$  for  $i = 1, 2, \dots, m-1$ , there is a non-trivial solution which satisfies  $\sum_{j=1}^n x_j = 1$  and this is treated as the  $m$ th equation of the system. This theorem is used here to solve the homogeneous systems.

Suppose that a solution was desired from the following linear programming problem.

		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$
250	$\geq$	2	3						
0	$\geq$	-1	2						
0	$\geq$			10	15	-20	-30		
0	$\geq$			2	4	-2	-4		
0	$\geq$	-1				1		1	
0	$\geq$		-1				1		1
$z(\min)$	$=$	5	4	-50	-60			-20	-25

where again profit in the  $z$  equation is negative cost. The necessary slacks are added and the problem assumes the form:



		$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$x_{10}$	$x_{11}$	$x_{12}$	$x_{13}$	$x_{14}$	$-z$
250	=	2	3							1						
0	=	-1	2								1					
0	=			10	15	-20	-30					1				
0	=			2	4	-2	-4						1			
0	=	-1				1		1						1		
0	=		-1				1		1						1	
0	=	5	4	-50	-60			-20	-25							1

Then if slacks are interspersed with the activities, and the constants column moved to the right side, the problem becomes:

$x_1$	$x_2$	$x_9$	$x_{10}$	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$	$x_7$	$x_8$	$x_{13}$	$x_{14}$	$-z$	Constants
2	3	1													= 250
-1	2		1												= 0
				10	15	-20	-30	1							= 0
				2	4	-2	-4		1						= 0
-1						1				1		1			= 0
	-1						1				1		1		= 0
5	4			-50	-60					-20	-25			1	= 0

This program represents many of the problems encountered in farm linear programming. The above program is easily decomposed to:



$$A_1 X_1 = b_1$$

$$A_2 X_2 = b_2$$

$$\bar{A}_1 X_1 + \bar{A}_2 X_2 + A_3 X_3 - P_0 z = \bar{b}$$

where  $A_1 X_1 = b_1$  represents  $\begin{bmatrix} 2 & 3 & 1 & - \\ -1 & 2 & - & 1 \end{bmatrix} X_1 = \begin{bmatrix} 250 \\ 0 \end{bmatrix}$

and  $A_2 X_2 = b_2$  represents  $\begin{bmatrix} 10 & 15 & -20 & -30 & 1 & - \\ 2 & 4 & -2 & -4 & - & 1 \end{bmatrix} X_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

and  $\bar{A}_1 X_1 + \bar{A}_2 X_2 + A_3 X_3 - P_0 z = \bar{b}$

represents  $\begin{bmatrix} -1 & - & - & - \\ - & -1 & - & - \\ 5 & 4 & - & - \end{bmatrix} X_1 + \begin{bmatrix} - & - & 1 & - & - & - \\ - & - & - & 1 & - & - \\ -50 & -60 & - & - & - & - \end{bmatrix} X_2 + \begin{bmatrix} 1 & - & 1 & - \\ - & 1 & - & 1 \\ -20 & -25 & - & - \end{bmatrix} X_3$   
 $- \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

From this the restricted master program is obtained. Again only the Phase II procedure is used since an initial feasible solution is obvious. The  $\bar{\Phi}_i$  and  $\bar{\mu}_j$  rows are used here instead of  $\Phi_i$  and  $\mu_j$  to consider homogeneous solutions. The restricted program then is:

Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	$\bar{\mu}_1$	Constants
$x_{13}$	1					0
$x_{14}$		1				0
$-z$			1			0
$\bar{\Phi}_1$				1		1
$\bar{\mu}_1$						1 (a)
					(b)	

The  $b_2 = 0$ , so only homogeneous solutions will result from the restrictions of the corresponding sub-program. This fact means all



the  $\delta_j^i = 0$ . Therefore the  $\bar{\mu}_1$  row ((a) row) may be dropped from consideration in the restricted master program. The  $\bar{\mu}_1$  column ((b) column) may also be left out since it is a trivial solution to  $\bar{A}_2 X_{21}$  where

$$X_{21} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The program is:

Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	Constants
$x_{13}$	1				0
$x_{14}$		1			0
$-z$			1		0
$\bar{\Phi}_1$				1	1

The column  $\bar{\Phi}_1$  represents the solution to  $\bar{A}_1 X_{11}$

where  $X_{11} = \begin{bmatrix} 0 \\ 0 \\ 250 \\ 0 \end{bmatrix}$  which is a non-homogeneous solution to  $A_1 X_1 = b_1$

and therefore  $\delta_1 = 1$  and  $\delta_1 \bar{\Phi}_1 = 1 \times 1 = 1$ . The multipliers are:

$$\pi^0 = [0 \ 0 \ 1]; \quad -s^0 = 0.$$

Therefore

$$\begin{aligned} \pi^0 \bar{A}_1 &= \begin{vmatrix} 5 & 4 & 0 & 0 \end{vmatrix} \\ \pi^0 \bar{A}_2 &= \begin{vmatrix} -50 & -60 & 0 & 0 & 0 & 0 \end{vmatrix} \\ \pi^0 A_3 &= \begin{vmatrix} -20 & -25 \end{vmatrix} \end{aligned}$$

The  $\pi^0 A_3$  values are read directly for placing in the program. That is to say that  $\pi^0 S_i - s^0$  and  $\pi^0 T_j$  must be more negative than -25 before the



column  $S_i$  or  $T_j$  will be entered into the matrix. Next the sub-programs are solved.

Solving sub-program  $L_1$ ,  $A_1 X_1 = b_1$

$$\text{Min. } (\pi \overline{A_1}) X_1 = z_1^1$$

or solve

$x_1$	$x_2$	$x_9$	$x_{10}$		Constants
2	3	1		=	250
-1	2		1	=	0
5	4	0	0	=	$z_1^1$

Again Phase II may be initiated directly as the slacks are in canonical form. Also there are no negative values in the  $z_1^1$  row so the solution

is optimal and is identical to  $X_{11}$ . Therefore the value of  $X_{12}$  is  $\begin{bmatrix} 0 \\ 0 \\ 250 \\ 0 \end{bmatrix}$

and  $z_1^1 = 0$ .

Next solve sub-program  $L_2$ , where  $A_2 X_2 = b_2$

$$\text{Min. } (\pi \overline{A_2}) X_1 = z_2^1$$

or solve

$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$		Constants
10	15	-20	-30	1		=	0
2	4	-2	-4		1	=	0
-50	-60	-	-	-	-	=	$z_2^1$



Transferring  $z_2^1$  to a column the program becomes:

$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$	$-z_2^1$		Constants
10	15	-20	-30	1			=	0
2	4	-2	-4		1		=	0
-50	-60					1	=	0

If this is solved, only trivial solutions of all  $x_i$ 's equal to zero result.

However there are solutions other than trivial solutions which exist.

This may be demonstrated if either  $x_5$  or  $x_6$  are forced to be constant

(say  $x_5 = 1$ ) and moved to the constants column. Then a feasible

solution other than a trivial solution may be obtained. However a more

logical approach to solving the problem is desired and this is the

procedure Dantzig<sup>14</sup> mentions on pages 136 - 138 of his text for solving

homogeneous systems.

The procedure necessitates adding another restriction that the  $\sum x_i = 1$ .

Therefore the sample sub-program becomes

$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$	$-z_2^1$		Constants
10	15	-20	-30	1			=	0
2	4	-2	-4		1		=	0
1	1	1	1	1	1		=	1
-50	-60					1	=	0

This may now be solved using standard linear programming procedure starting with Phase I, since there are no variables in canonical form. The simplex



procedure using multipliers is again used to obtain the solution program:

Basis Variable	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$	Artificials			$-z_2^1$	$-w$	Constants
	$x_{a1}$	$x_{a2}$	$x_{a3}$									
$x_{a1}$	10	15	-20	-30	1		1				=	0
$x_{a2}$	2	4	-2	-4		1		1			=	0
$x_{a3}$	1	1	1	1	1	1			1		=	1
$-z_2^1$	-50	-60								1	=	0
$-w$	-13	-20	21	33	-2	-2					1 =	-1

Solution:

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^1$	$-w$	Constants	Variable Entering
$x_{a1}$	1					0	$x_4$ 15
$x_{a2}$		1				0	3
$x_{a3}$			1			1	1
$-z_2^1$				1		0	-60
$-w$					1	-1	-20

Multipliers:  $[0 \ 0 \ 0 \ 0 \ 1]$

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	= -23	-26	35	38	-2	-2



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^1$	$-w$	Constants	Variable Entering
$x_4$	.067					0	$x_6$ -2
$x_{a2}$	-.253	1				0	<span style="border: 1px solid black;">4</span>
$x_{a3}$	-.067		1			1	3
$-z_2^1$	4			1		0	-120
$-w$	1.333				1	-1	-7

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	= .333	0	-5.667	-7.000	-.667	-2.000

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^1$	$-w$	Constants	Variable Entering
$x_4$	-.067	.500				0	$x_{11}$ -.067
$x_6$	-.067	.250				0	-.067
$x_{11}$	.133	-.750	1			1	<span style="border: 1px solid black;">1.133</span>
$-z_2^1$	-4	30		1		0	-4
$-w$	.867	1.750			1	-1	-1.133

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	= -.833	0	.167	0	-1.133	-.250



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^1$	-w	Constants
$x_4$	-.059	.456	.059			.059
$x_6$	-.059	.176	.059			.059
$x_{11}$	.118	-.662	.882			.882
$-z_2^1$	-3.527	27.353	3.527	1		3.527
-w	1	1	1		1	0

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	= 0	0	0	0	0	0

End of Phase I and begin Phase II. The multipliers become the numbers found in the  $-z_2^1$  row as the -w row and the -w column may be dropped.

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^1$	Constants	Variable Entering
$x_4$	-.059	.456	.059		.059	$x_3$ <u>.382</u>
$x_6$	-.059	.176	.059		.059	-.133
$x_{11}$	.133	-.662	.882		.882	.724
$-z_2^1$	-3.527	27.353	3.527	1	3.527	-27.059

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{c}_j$	= -27.059	0	19.412	0	0	30.882



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^1$	Constants
$x_3$	-.154	1.192	.154		.154
$x_6$	-.077	.323	.077		.077
$x_{11}$	.231	-1.538	.769		.769
$-z_2^1$	-7.692	59.615	7.692	1	7.692

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{c}_j$	= 0	70.769	42.308	0	0	67.308

All the  $\bar{c}_j$  values are positive so the sub-program is optimum. The solution is  $x_3 = .154$ ,  $x_6 = .077$ ,  $x_{11} = .769$ ,  $x_4 = x_5 = x_{12} = 0$ ,  $z_2^1 = -7.692$ . Some explanation of the small numerical discrepancies which occur is required here. All these programs were worked in fractions and then changed to decimal values. Therefore round-off error appears. Normally these problems would be worked in decimal values since

a) the data is usually in decimal notation

b) the fractions become unmanageable with larger programs.

The above solution gives a non-trivial solution to enter in the restricted

master program. The column  $X_{22}$  is  $\begin{bmatrix} .154 \\ 0 \\ 0 \\ .077 \\ .769 \\ 0 \end{bmatrix}$  and  $\bar{A}_2 X_{22} = \begin{bmatrix} 0 \\ .077 \\ -7.692 \end{bmatrix}$

This column is compared to other choices for entering columns; namely

$\bar{A}_1 X_{12}$  and  $A_3$ . Therefore comparisons are made between the following columns:



$$a) \bar{A}_1 X_{12} = \bar{A}_1 X_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

b)  $A_3$ , which has four columns in it, is associated with variables

$$x_7, x_8, x_{13}, \text{ and } x_{14}. \text{ They are } \begin{bmatrix} 1 \\ 0 \\ -20 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -25 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

The latter two columns are already in the basis, so are ruled out of the present comparison.

$$c) \bar{A}_2 X_{22} = \begin{bmatrix} 0 \\ .077 \\ -7.692 \end{bmatrix} \text{ which is a homogeneous solution and therefore}$$

multiples of this solution may be considered.

The column which qualifies to enter the basis from the  $A_3$  matrix is the one with the most negative value when multiplied by the row vector  $\pi^0$ . The most negative value is compared with  $(\pi^0 \bar{A}_1) X_{1i}$  -s or  $(\pi^0 \bar{A}_2) X_{2j}$ . The most negative of these values are chosen in the normal procedure. However if a homogeneous solution is found, then multiples of this solution are also possible and there are no multiplier values associated with the column. Thus any homogeneous solution with a positive -z value in the sub-program means the transformed column has a negative -z row entry in the restricted master program. Due to the possibility of multiples of the column, it can be made the most negative column. Therefore homogeneous solutions with negative -z row entries in the restricted master program are preferred to other solutions entering the basis. In the sample problem, the homogeneous solution  $\begin{bmatrix} 0 \\ .077 \\ -7.692 \end{bmatrix}$  enters. This has no  $\bar{\Phi}_i$  restriction associated with it, so the entry in the  $\bar{\Phi}_i$  row is zero. In the case of more than one sub-program resulting in homogeneous solutions, the choice is arbitrary.



The restricted master program is illustrated below where the symbol  $\bar{\mu}_j$  means multiples of homogeneous solutions from

$$\begin{aligned} A_2 X_2 &= b_2 \\ \text{Min. } (\pi \bar{A}_2) X_2 &= z_2^1 \end{aligned}$$

Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	Constants	Entering Variable
						$\bar{\mu}_2$
$x_{13}$	1				0	0
$x_{14}$		1			0	<span style="border: 1px solid black;">.077</span>
$-z$			1		0	-7.962
$\bar{\Phi}_1$				1	1	0

Solved, it becomes:

Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	Constants
$x_{13}$	1				0
$\bar{\mu}_2$		13			0
$-z$		100	1		0
$\bar{\Phi}_1$				1	1

The process now repeats itself. The multipliers are:  $\pi^1 = [0 \ 100 \ 1]$ ;  $s^1 = 0$ .

Therefore

$$\begin{aligned} \pi^1 \bar{A}_1 &= [5 \ -96 \ 0 \ 0] \\ \pi^1 \bar{A}_2 &= [-50 \ -60 \ 0 \ 100 \ 0 \ 0] \\ \pi^1 A_3 &= [-20 \ 75 \ 0 \ 100] \end{aligned}$$

Most negative of  $\pi^1 A_3$  is -20 which is the entry in the  $-z$  row of restricted master program for variable  $x_7$ . This value of -20 is saved for future



comparison.

$$\begin{aligned} \text{Solving } L_1: \quad & A_1 X_1 = b_1 \\ \text{Min. } (\pi^1 A_1) X_1 &= z_1^2 \end{aligned}$$

Program

$x_1$	$x_2$	$x_9$	$x_{10}$	$-z_1^2$	Constants
2	3	1			250
-1	2		1		0
5	-96	0	0	1	0

Solution: Proceed directly to Phase II.

Basis Variable	$x_9$	$x_{10}$	$-z_1^2$	Constants	Variable Entering
$x_9$	1			250	$x_2$ 3
$x_{10}$		1		0	<span style="border: 1px solid black;">2</span>
$-z_1^2$			1	0	-96

	$x_1$	$x_2$	$x_9$	$x_{10}$
$\bar{c}_j$	= 5	-96	0	0



Basis Variable	$x_9$	$x_{10}$	$-z_1^2$	Constants	Entering Variable
$x_9$	1	-1.500		250	$x_1$ [3.5]
$x_2$		.500		0	-.5
$-z_1^2$		48	1	0	-43

	$x_1$	$x_2$	$x_9$	$x_{10}$
$\bar{c}_j$	= -43	0	0	48

Basis Variable	$x_9$	$x_{10}$	$-z_1^2$	Constants
$x_1$	.286	-.429		71.429
$x_2$	.143	.286		35.715
$-z_1^2$	12.286	29.556	1	3071.429

	$x_1$	$x_2$	$x_9$	$x_{10}$
$\bar{c}_j$	= 0	0	12.286	29.556

Solution:  $z_1^2 = -3071.429$ ,  $X_{13} = \begin{bmatrix} 71.429 \\ 35.715 \\ 0 \\ 0 \end{bmatrix}$ ,

$z_1^2 - s^1 = -3071.429 - 0 = -3071.429.$

Solving  $L_2$ :  $A_2 X_2 = b_2$   
 $\text{Min}(\pi^1 A_2) X_2 = z_2^2$

which in matrix form is



$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$	$-z_2^2$		Constants
10	15	-20	-30	1			=	0
2	4	-2	-4		1		=	0
-50	-60		100			1	=	0

Since this is again a problem where  $b_2 = 0$ , another restriction of  $\sum_{i=1}^n x_i = 1$  must be added and Phase I initiated to solve the problem.

$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	$-w$	Constants
10	15	-20	-30	1		1					0
2	4	-2	-4		1		1				0
1	1	1	1	1	1			1			1
-50	-60		100						1		0
-13	-20	21	33	-2	-2					1	-1

### Solution:

Phase I: Again the artificial variables form the initial basis.

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	$-w$	Constants	Entering Variable
$x_{a1}$	1					0	$x_4$ 15
$x_{a2}$		1				0	4
$x_{a3}$			1			1	1
$-z_2^2$				1		0	-60
$-w$					1	-1	-20

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	-13	-20	21	33	-2	-2



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	$-w$	Constants	Entering Variable
							$x_6$
$x_4$	.0677					0	-2
$x_{a2}$	-.253	1				0	<span style="border: 1px solid black;">4</span>
$x_{a3}$	-.067		1			1	3
$-z_2^2$	4			1		0	-20
$-w$	1.333				1	-1	-7

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	= .333	0	-5.667	-7.000	-.667	-2.000

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	$-w$	Constants	Entering Variable
							$x_{11}$
$x_4$	-.067	.500				0	-.067
$x_6$	-.067	.250				0	-.067
$x_{a3}$	.133	-.750	1			1	<span style="border: 1px solid black;">1.133</span>
$-z_2^2$	2.667	5		1		0	2.667
$-w$	.867	1.750			1	-1	-1.133

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	= -.833	0	.167	0	-1.133	-.250



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	$-w$	Constants
$x_4$	-.059	.456	.059			.059
$x_6$	-.059	.206	.059			.059
$x_{11}$	-.118	-.662	.882			.882
$-z_2^2$	2.353	6.765	-2.353	1		-2.353
$-w$	1	1	1		1	0

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	= 0	0	0	0	0	0

End of Phase I. Start Phase II.

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	Constants	Entering Variable
$x_4$	-.059	.456	.059		.059	$x_5$ .324
$x_6$	-.059	.206	.059		.059	<u>.824</u>
$x_{11}$	.118	-.662	.882		.882	-.147
$-z_2^2$	2.353	6.765	-2.353	1	-2.353	-62.941

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{c}_j$	= -15.294	0	-62.941	0	0	4.412



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	Constants	Entering Variable
$x_4$	-.050	.375	.036		.036	$x_3$ <span style="border: 1px solid black;">.429</span>
$x_5$	-.071	.250	.071		.071	-.143
$x_{11}$	.107	.625	.893		.893	.714
$-z_2^2$	-2.143	22.50	2.143	1	2.143	-24.286

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{c}_j$	= -24.286	0	0	219.286	0	2.143

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^2$	Constants
$x_4$	-.083	.875	.083		.083
$x_5$	-.083	.375	.083		.083
$x_{11}$	.167	-1.25	.833		.833
$-z_2^2$	-4.167	43.750	4.167	1	4.167

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{c}_j$	= 0	56.667	0	54.167	0	47.917

Solution:  $X_{23} = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} .083 \\ 0 \\ .083 \\ 0 \\ .833 \\ 0 \end{bmatrix} ; z_2^2 = -4.167$

The choice is now a matter of examining the solutions to the sub-programs for the most negative. From the  $A_3$  matrix the value of the column with the most negative value is -20; from  $A_1 X_1 = b_1$ ;  
 $\text{Min}(\pi \bar{A}_1) X_1 = z_1^2$



$z_1^2 - s^1 = -71.429$ ; and the homogeneous solution has a negative value of  $z_2^2 = -4.167$ . Therefore the homogeneous solution is the solution to enter the basis of the restricted master program. Then  $\bar{A}_2 X_{23} = \begin{bmatrix} .083 \\ 0 \\ -4.167 \end{bmatrix}$

and the column has no  $\bar{\Phi}_1$  restriction and can then be modified to fit the present basis. Therefore the program becomes:

Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	Constants	Entering Variable
$x_{13}$	1				0	$\bar{\mu}_3$ .833
$\bar{\mu}_2$		13			0	0
$-z$		100	1		0	-4.167
$\bar{\Phi}_1$				1	1	0

Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	Constants
$\bar{\mu}_3$	12				0
$\bar{\mu}_2$		13			0
$-z$	50	100	1		0
$\bar{\Phi}_1$				1	1

The multipliers are:  $\pi^2 = [50 \ 100 \ 1]$ ;  $s^2 = 0$

Therefore:

$$\pi^2 \bar{A}_1 = [-45 \ -96 \ 0 \ 0]$$

$$\pi^2 \bar{A}_2 = [-50 \ -60 \ 50 \ 100 \ 0 \ 0]$$

$$\pi^2 \bar{A}_3 = [30 \ 75 \ 50 \ 100]$$



All entries in the  $-z$  row of the restricted master program for the columns of matrix  $A_3$  are positive so these can be ignored for determining the next column to enter. Next solve

$$\begin{aligned} A_1 X_1 &= b_1 \\ \text{Min}(\pi^2 A_1) X_1 &= z_1^3 \end{aligned}$$

Program

$x_1$	$x_2$	$x_9$	$x_{10}$	$-z_1^3$	Constants
2	3	1			250
-1	2		1		0
-45	-96			1	0

Solution: Proceed directly to Phase II.

Basis Variable	$x_9$	$x_{10}$	$-z_1^3$	Constants	Entering Variable
$x_9$	1			25	$x_2$
$x_{10}$		1		0	$\boxed{2}$
$-z_1^3$			1	0	-96

	$x_3$	$x_4$	$x_9$	$x_{10}$
$c_j$	-45	-96	0	0



Basis Variable	$x_9$	$x_{10}$	$-z_1^3$	Constants	Entering Variable
$x_9$	1	-1.500		250	$x_1$ 3.500
$x_2$		.500		0	-0.500
$-z_1^3$		48	1	0	-93

	$x_1$	$x_2$	$x_9$	$x_{10}$
$\bar{c}_j$	= -93	0	0	48

Basis Variable	$x_9$	$x_{10}$	$-z_1^3$	Constants
$x_1$	.286	-.429		71.429
$x_2$	.143	.286		35.715
$-z_1^3$	26.571	8.143	1	6642.857

	$x_1$	$x_2$	$x_9$	$x_{10}$
$\bar{c}_j$	= 0	0	26.571	8.143

Solution:  $X_{14} = \begin{bmatrix} 71.429 \\ 35.715 \\ 0 \\ 0 \end{bmatrix}$ ;  $z_1^3 = -6642.857$ ;  $z_1^3 - s^1 = -6642.857$ .

Solving:

$$A_2 X_2 = b_2$$

$$\text{Min}(\pi^2 A_2) X_2 = z_2^3$$

Problem:

$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^3$	-w	Constants
10	15	-20	-30	1		1					0
2	4	-2	-4		1		1				0
1	1	1	1	1	1			1			1
-50	-60	50	100						1		0
-13	-20	21	33	-2	-2					1	-1



Solution: Phase I

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^3$	$-w$	Constants	Entering Variable
$x_{a1}$	1					0	$x_4$ <span style="border: 1px solid black;">15</span>
$x_{a2}$		1				0	4
$x_{a3}$			1			1	1
$-z_2^3$				1		0	-60
$-w$					1	-1	-20

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	-13	-20	21	33	-2	-2

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^3$	$-w$	Constants	Entering Variable
$x_4$	.067					0	$x_6$ -2
$x_{a2}$	-.253	1				0	<span style="border: 1px solid black;">4</span>
$x_{a3}$	-.067		1			1	3
$-z_2^3$	4			1		0	-20
$-w$	1.333				1	-1	-7

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	.333	0	-5.667	-7.00	-.667	-2.00



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^3$	$-w$	Constants	Entering Variable
							$x_{11}$
$x_4$	-.067	.500				0	-.067
$x_6$	-.067	.250				0	-.067
$x_{a3}$	.133	-.750	1			1	1.133
$-z_2^3$	2.667	5		1		0	2.667
$-w$	.867	1.750			1	-1	-1.133

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	-.833	0	.167	0	-1.133	-.250

Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^3$	$-w$	Constants	Entering Variable
							$x_3$
$x_5$	-.059	.456	.059			.059	.382
$x_6$	-.059	.206	.059			.059	-.118
$x_{11}$	.118	-.662	.882			.882	.735
$-z_2^3$	2.353	6.765	-2.353	1		-2.353	-15.294
$-w$	1	1	1		1	0	

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{d}_j$	0	0	0	0	0	0

Phase II

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{c}_j$	-15.294	0	-12.941	0	0	4.412



Basis Variable	$x_{a1}$	$x_{a2}$	$x_{a3}$	$-z_2^3$	Constants
$x_3$	-.154	1.192	.154		.154
$x_6$	-.154	.346	.077		.077
$x_{11}$	.231	-1.538	.769		.769
$-z_2^3$	0	25	0	1	0

	$x_3$	$x_4$	$x_5$	$x_6$	$x_{11}$	$x_{12}$
$\bar{c}_j$	= 0	40	0	0	0	25

The solution is obtained here and the value of  $z_2^3 = 0$ . Therefore this solution will add nothing to the profit even when multiplied and we drop it from consideration for the present. Therefore the entering column is the column associated with  $A_1 X_1 = b_1$  which is  $\bar{A}_1 X_{14} = \begin{bmatrix} -71.429 \\ -35.715 \\ 500 \end{bmatrix}$   
 $\text{Min}(\pi^2 \bar{A}_1) X_1 = z_1^3$

Since this column is restricted by  $\bar{\Phi}_i$ , another entry of 1 in the  $\bar{\Phi}_i$  restriction is necessary. The column before modifying to the present inverse is  $\begin{bmatrix} -71.429 \\ -35.715 \\ 500 \\ 1 \end{bmatrix}$ .

Solving:

Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	Constants	Entering Variable
$\bar{\mu}_3$	12				0	$\bar{\Phi}_4$ -857.143
$\bar{\mu}_2$		13			0	-464.286
$-z$	50	100	1		0	-6642.857
$\bar{\Phi}_1$				1	1	<span style="border: 1px solid black;">1</span>



Basis Variable	$x_{13}$	$x_{14}$	$-z$	$\bar{\Phi}_1$	Constants
$\bar{\mu}_3$	12		857.143		857.143
$\bar{\mu}_2$		13	464.286		464.286
$-z$	50	100	1 6642.857		6642.857
$\bar{\Phi}_4$				1	1

Again the process is repeated.  $\pi^3 = 50 \ 100 \ 1$ ;  $s^3 = -6642.857$ . However  $\pi^3 = \pi^2$ . Therefore the solutions for the two sub-programs are the same for this step as the last step and for this reason the columns in  $A_3$  may be disregarded; the homogeneous solutions to  $A_2 X_2 = b_2$   $\text{Min}(\pi^3 \bar{A}_2) X_2 = z_2^4$  may be disregarded, as the optimum does not increase whatever the multiple may be; and the solution to  $A_1 X_1 = b_1$  is the same as the solution to  $\text{Min}(\pi^3 \bar{A}_1) X_1 = z_1^4$   $A_1 X_1 = b_1$ . The value of  $z_1^3 = z_1^4 = -6642.857$ . Now the only  $\text{Min}(\pi^2 \bar{A}_1) X_1 = z_1^3$  difference between the steps shows up in finding

$$z_1^4 - s^3 = -6642.857 - (-6642.857) = 0.$$

This indicates that the optimum has been reached for the master program and the solution is

$$X_{22} \bar{\mu}_2 + X_{23} \bar{\mu}_3, X_{14} \bar{\Phi}_4, \text{ and } X_3, \text{ where } X_3 = 0;$$

$$\bar{\mu}_2 = 464.29; \bar{\mu}_3 = 857.14; \bar{\Phi}_4 = 1;$$



$$X_{22} = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} .154 \\ 0 \\ 0 \\ .077 \\ .769 \\ 0 \end{bmatrix}; \quad X_{23} = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} .083 \\ 0 \\ .083 \\ 0 \\ .833 \\ 0 \end{bmatrix}; \quad X_{14} = \begin{bmatrix} x_1 \\ x_2 \\ x_9 \\ x_{10} \end{bmatrix} = \begin{bmatrix} 71.43 \\ 35.71 \\ 0 \\ 0 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} x_7 \\ x_8 \\ x_{13} \\ x_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Then:

$$X_{22} \bar{\mu}_2 = \begin{bmatrix} .154 \\ 0 \\ 0 \\ .077 \\ .769 \\ 0 \end{bmatrix} 464.29 = \begin{bmatrix} 71.43 \\ 0 \\ 0 \\ 35.71 \\ 357.14 \\ 0 \end{bmatrix}$$

$$X_{23} \bar{\mu}_3 = \begin{bmatrix} .083 \\ 0 \\ .083 \\ 0 \\ .833 \\ 0 \end{bmatrix} 857.14 = \begin{bmatrix} 71.43 \\ 0 \\ 71.43 \\ 0 \\ 714.29 \\ 0 \end{bmatrix}$$

Therefore:

$$x_3 = 71.43 + 71.43 = 142.86$$

$$x_4 = 0 + 0 = 0$$

$$x_5 = 0 + 71.43 = 71.43$$

$$x_6 = 35.71 + 0 = 35.71$$

$$x_{11} = 357.14 + 714.29 = 1071.43$$

$$x_{12} = 0 + 0 = 0$$

$$\begin{bmatrix} X_{13} \end{bmatrix} 1 = \begin{bmatrix} 71.43 \\ 35.71 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_1 &= 71.43 \\ x_2 &= 35.71 \\ x_9 &= 0 \\ x_{10} &= 0 \end{aligned}$$

$$X_3 = 0, \text{ so } x_7 = 0; x_8 = 0; x_{13} = 0; x_{14} = 0.$$



In summary then,  $x_1 = 71.43$ ;  $x_2 = 35.71$   $x_3 = 142.86$ ;  $x_5 = 71.43$ ;  
 $x_6 = 35.71$ ;  $x_{11} = 1071.43$ .

$$x_4 = x_7 = x_8 = x_9 = x_{10} = x_{12} = x_{13} = x_{18} = 0.$$

$$z = -6642.86.$$

With the development of the procedure for homogeneous solutions, this means decomposition will work on any program which will solve without being decomposed. All that remains is to develop a Phase I procedure to start the program.

### 7.1.3 Development of the Phase I Procedure.

If there is no readily available basic feasible solution (i.e. there are no columns in the linking matrices which have variables in canonical form), it is necessary to use a procedure to obtain an initial feasible solution. To do this, it is necessary to first add artificial variables and another row for variables. As well, one arbitrary solution from each sub-program is required to fill in the starting basis of the master program. This basis is:

$$S_1 \Phi_1 + T_1 \mu_1 - P_0 z + U_1 E_1 + U_2 E_2 \dots \dots \dots + U_m E_m = \bar{b}$$

$$\Phi_1 = 1$$

$$\mu_1 = 1$$

$$E_1 + E_2 \dots \dots \dots + E_m - w = 0$$

where  $U_1$  is an  $m + 1$  component unit vector with 1 in row  $i$  and zeros elsewhere. The sign of the one is chosen so that  $E_i$  is positive. This can be arranged to look as below:



$$\begin{array}{rcl}
 + U_1 E_1 & + & U_2 E_2 \dots\dots\dots + U_m E_m \quad -P_0 z + S_1 \Phi_1 + T_1 \mu_1 = \bar{b} \\
 E_1 & + & E_2 \dots\dots\dots + E_m \quad -w = 0 \\
 & & \Phi_1 = 1 \\
 & & \mu_1 = 1
 \end{array}$$

The variables in the initial basis are the artificial variables  $E_1$ ---  $E_m$ ,  $-w$ ,  $-z$ ,  $\Phi_1$ , and  $\mu_1$ . The initial coefficients of the columns for variables not in the basis are calculated from the basis variables using row multipliers ( $\sigma'$ ,  $f$ , and  $g$ ) which will make the basis variable price out to zero in the  $-w$  row except the  $-w$  column which prices out to one.

Here the basis is:

$$\left[ \begin{array}{ccccccc}
 + U_1 E_1 & + & U_2 E_2 & \dots\dots\dots & + & U_m E_m & -P_0 z + S_1 \Phi_1 + T_1 \mu_1 \\
 E_1 & + & E_2 & \dots\dots\dots & + & E_m & -w \\
 & & & & & & \Phi_1 \\
 & & & & & & \mu_1
 \end{array} \right] \begin{array}{l} \sigma' \\ 1 \\ f \\ g \end{array}$$

The  $\sigma'$  multipliers (Phase I multipliers are usually labelled  $\sigma$ ) are of the form  $+1_1, +1_2 \dots\dots +1_m, 0 \quad 1$  and are used to derive the functional rows for the sub-programs by multiplying with the  $\bar{A}_i$  matrices. This could be repeated for every iteration by looking at each basis, but is inconvenient. Therefore the initial functional coefficients are calculated and are placed on the bottom of the  $\bar{A}_i$  matrices to make up the  $\bar{A}_i'$  matrices. The row multipliers used for subsequent iterations may be obtained from the inverse used for solving the problem and are used to modify the initial coefficients calculated from the first basis. They are the usual Phase I multipliers and may be labelled  $\sigma$ .



Mathematically:

$$\begin{array}{rcl} \sigma^i \bar{A}_i & = & \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} \\ \bar{A}_i & = & \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \dots & \bar{A}_{1n} \\ \bar{A}_{21} & \bar{A}_{22} & \dots & \bar{A}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{A}_{m1} & \bar{A}_{m2} & \dots & \bar{A}_{mn} \end{bmatrix} \\ \sigma^i \bar{A}_i & = & \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} \\ \sigma^i & = & \begin{bmatrix} \sigma_1^i & \sigma_2^i & \dots & \sigma_m^i & 0 & 1 \end{bmatrix} \end{array} \quad \left. \vphantom{\begin{array}{rcl} \sigma^i \bar{A}_i \\ \bar{A}_i \\ \sigma^i \bar{A}_i \\ \sigma^i \end{array}} \right\} \bar{A}_i'$$

Also let the initial multipliers required for the  $\Phi$  row and  $\mu$  row be  $f$  and  $g$  respectively. (i.e. In order to make the column for  $\Phi_1$  price-out to zero, a multiplier,  $f$  must be used on the  $\Phi$  row. Similarly the multiplier,  $g$ , is required for the  $\mu$  row). If the entry in the  $-w$  row of the  $\Phi_1$  column is labelled  $(-s')$ , and the entry in the  $-w$  row of the  $\mu_1$  column is labelled  $(-t')$ , the criteria for determining when the first basic feasible solution occurs is where all

$$\begin{array}{l} \sigma \bar{A}_1' X_{1i} + (-s') + f \geq 0 \\ \sigma \bar{A}_2' X_{2j} + (-t') + g \geq 0 \\ \sigma A_3' \geq 0 \quad \text{and } w = 0. \end{array}$$

The objective of course, is to minimize  $w$ . No feasible solution exists if  $w$  is positive when all possible entering columns have been exhausted. Briefly stated, the procedure is as follows:

- 1) Use the  $\sigma^i$  multipliers to calculate the  $a_i$  values.
- 2) Determine the inverse of the current basis.
- 3) Use the inverse to calculate the corresponding constants column.
- 4) Use the  $\sigma$  multipliers obtained by examining the  $-w$  row to determine functional rows for the sub-programs.
- 5) Solve the sub-programs and obtain the linear transform.
- 6) Find the column which has the most negative value of  $\sigma \bar{A}_1' X_{1i} + (-s') + f$ ,  $\sigma \bar{A}_2' X_{2j} + (-t') + g$ , or  $\sigma A_3'$ .



- 7) The column determined above has a one placed in the  $\Phi$  or  $\mu$  row and zero in the remaining position. The entry in the  $-w$  row is  $\bar{A}_1' X_{1i} + f$  or  $\bar{A}_2' X_{2j} + g$  as the case may be. If the column comes from  $A_3'$ , it is used directly.
- 8) Alter the column to fit the inverse by multiplying the rows of the inverse by the entering column to get the new row entries of the entering column.
- 9) Enter the column in the basis using the normal pivotal row procedure.
- 10) Repeat until minimum  $w = 0$ , or no feasible solution exists.
- 11) If minimum  $w = 0$ , remove  $-w$  row and  $-w$  column and follow the normal decomposition procedure outlined previously.

An example will illustrate. Suppose the first sample problem used to illustrate the general decomposition technique was not started at a basic feasible solution. Rather suppose that

$$S_1 = \begin{bmatrix} 10 \\ 10 \\ -20 \end{bmatrix}; \quad T_1 = \begin{bmatrix} 20 \\ 31.25 \\ -65 \end{bmatrix}$$

Then:

$$\begin{aligned} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} E_1 + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} E_2 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (-z) &+ \begin{bmatrix} 10 \\ 10 \\ -20 \end{bmatrix} 1 + \begin{bmatrix} 20 \\ 31.25 \\ -65 \end{bmatrix} 1 = \begin{bmatrix} 20 \\ 20 \\ 0 \end{bmatrix} \\ E_1 + E_2 + 1 (-w) &= 0 \\ &1 = 1 \\ &1 = 1 \end{aligned}$$

Then:

$$E_1 = 10; E_2 = 21.25; -z = 85; -w = 31.25$$



The problem becomes, in numerical terms,

							Multipliers						
$\begin{bmatrix} -10 \\ - \\ - \\ 10 \\ - \\ - \end{bmatrix}$	+	$\begin{bmatrix} - \\ -21.25 \\ - \\ 21.25 \\ - \\ - \end{bmatrix}$	+	$\begin{bmatrix} - \\ - \\ 85 \\ - \\ - \\ - \end{bmatrix}$	+	$\begin{bmatrix} - \\ - \\ - \\ -31.25 \\ - \\ - \end{bmatrix}$	+	$\begin{bmatrix} 10 \\ 10 \\ -20 \\ - \\ 1 \\ - \end{bmatrix}$	+	$\begin{bmatrix} 20 \\ 31.25 \\ -65 \\ - \\ - \\ 1 \end{bmatrix}$	=	$\begin{bmatrix} 20 \\ 20 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$	1 1 0 1 -20 -51.25

The multipliers required to price out to zero all column entries in the -w row except the entry for the -w column, are listed along the right side of the requirements.

$$\sigma' = [1 \ 1 \ 0 \ 1]; f = -20; g = -51.25$$

Using the  $\sigma'$  multipliers to calculate the  $\bar{A}_i'$  matrices and  $A_3'$  matrix where

$$\bar{A}_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & .5 & 0 & 0 \\ -2 & -3 & 0 & 0 \end{bmatrix}; \quad \bar{A}_2 = \begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \end{bmatrix}; \quad A_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

we obtain

$$\sigma' \bar{A}_1 = [2 \ 1.5 \ 0 \ 0]; \quad \sigma' \bar{A}_2 = [3 \ 1.25 \ 0 \ 0]$$

and

$$\sigma' A_3 = [1 \ 1]$$

and finally

$$\bar{A}_1' = \begin{bmatrix} 1 & 1 & - & - \\ 1 & .5 & - & - \\ -2 & -3 & - & - \\ 2 & 1.5 & - & - \end{bmatrix}; \quad \bar{A}_2' = \begin{bmatrix} 1 & 1 & - & - \\ 2 & .25 & - & - \\ -4 & -1 & - & - \\ 3 & 1.25 & - & - \end{bmatrix}; \quad \text{and } A_3' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$



The basis matrix is:

$$\begin{bmatrix} -10 & - & - & - & 10 & 20 \\ - & -21.25 & - & - & 10 & 31.25 \\ - & - & 85 & - & -20 & -65 \\ - & - & - & 1 & - & - \\ - & - & - & - & 1 & - \\ - & - & - & - & - & 1 \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 0 \\ -31.25 \\ 1 \\ 1 \end{bmatrix}$$

Next the inverse of the basis must be calculated. There are several ways of doing this, but a simple method follows. If the inverse of

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$$

is desired, it may be obtained by supplementing the

equations with  $y_i$  in canonical form as follows:

$$a_{11}x_1 + a_{12}x_2 + y_1 = 0$$

$$a_{21}x_1 + a_{22}x_2 + y_2 = 0$$

Then the  $x_i$  are placed in canonical form by algebraic manipulation:

$$x_1 + B_{11}y_1 + B_{12}y_2 = 0$$

$$x_2 + B_{21}y_1 + B_{22}y_2 = 0$$

Therefore if one needs to find the inverse of  $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$  it could be

calculated using the above method and would be the matrix  $\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$

In the problem, the coefficient matrix of the basis variables is (the variable is listed at the top of the column)



$E_1$	$E_2$	$-z$	$-w$	$\Phi_1$	$\mu_1$
-1	0	0	0	10	20
0	-1	0	0	10	31.25
0	0	1	0	-20	-65
0	0	0	1	0	0
0	0	0	0	1	0
0	0	0	0	0	1

and it is required that the inverse of this matrix is obtained.

Then

$$\begin{array}{rcll}
 -1E_1 & + 10\Phi_1 + 20\mu_1 & + y_1 & = 0 \\
 -1E_2 & + 10\Phi_1 + 31.25\mu_1 & y_2 & = 0 \\
 1(-z) & - 20\Phi_1 - 65\mu_1 & y_3 & = 0 \\
 1(-w) & & y_4 & = 0 \\
 1\Phi_1 & & y_5 & = 0 \\
 1\mu_1 & & y_6 & = 0
 \end{array}$$

$$\begin{array}{rcll}
 E_1 & - 1y_1 & + 10y_5 + 20y_6 & = 0 \\
 E_2 & - 1y_2 & + 10y_5 + 31.25y_6 & = 0 \\
 (-z) & + 1y_3 & + 20y_5 + 65y_6 & = 0 \\
 (-w) & + 1y_4 & & = 0 \\
 \Phi_1 & + 1y_5 & & = 0 \\
 \mu_1 & + 1y_6 & & = 0
 \end{array}$$



and the inverse is:

$$\begin{bmatrix} -1 & - & - & - & 10 & 20 \\ - & -1 & - & - & 10 & 31.25 \\ - & - & 1 & - & 20 & 65 \\ - & - & - & 1 & - & - \\ - & - & - & - & 1 & - \\ - & - & - & - & - & 1 \end{bmatrix}$$

The constants column corresponding to the basis is determined by multiplying the row in the inverse by the initial constants column for the corresponding row entry in the new constants column. In numerical terms, it is:

$$\begin{bmatrix} -1 & & & & & \\ & -1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} 10 & 20 \\ 10 & 31.25 \\ 20 & 65 \\ -31.25 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 21.25 \\ 85 \\ -31.25 \\ 1 \\ 1 \end{bmatrix}$$

Therefore our program to begin minimizing -w becomes:

Basis Variable	E <sub>1</sub>	E <sub>2</sub>	-z	-w	Φ <sub>1</sub>	μ <sub>1</sub>	Constants
E <sub>1</sub>	-1				10	20	10
E <sub>2</sub>		-1			10	31.25	21.25
-z			1		20	65	85
-w				1			-31.25
Φ <sub>1</sub>					1		1
μ <sub>1</sub>						1	1



$$\sigma_a = [0 \ 0 \ 0 \ 1]; \quad -s' = 0; \quad -t' = 0; \quad \sigma_a \bar{A}_1' = [2 \ 1.5 \ 0 \ 0];$$

$$\sigma_a \bar{A}_2' = [3 \ 1.25 \ 0 \ 0]; \quad \sigma_a A_3' = [1 \ 1]$$

The sub-programs are:

$$L_1 = \begin{array}{l} A_1 X_1 = b_1 \\ \text{Min. } (\sigma_a \bar{A}_1') X_1 = w_1^1 \end{array} \quad \text{and} \quad L_2 = \begin{array}{l} A_2 X_2 = b_2 \\ \text{Min. } (\sigma_a \bar{A}_2') X_2 = w_2^1 \end{array}$$

For the first sub-program,  $L_1$ , this means:

$$\begin{bmatrix} 1 & 1 & 1 & - \\ - & 2 & - & 1 \end{bmatrix} X_1 = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\text{Min. } \begin{bmatrix} 2 & 1 & 5 & 0 \end{bmatrix} X_1 = w_1^1$$

which in linear programming form is:

$x_1$	$x_2$	$x_5$	$x_6$	Constants
1	1	1		10
	2		1	5
2	1.5	0	0	$w_1^1$

The solution is  $X_{12} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 5 \end{bmatrix}$

Similarly the sub-program  $L_2$  becomes:

$x_3$	$x_4$	$x_7$	$x_8$	Constants
1	1	1		20
	1		1	5
3	1.25	0	0	$w_2^1$



The solution is  $X_{22} = \begin{bmatrix} 0 \\ 0 \\ 20 \\ 5 \end{bmatrix}$

Since  $X_{12} = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 5 \end{bmatrix}$   $X_{22} = \begin{bmatrix} 0 \\ 0 \\ 20 \\ 5 \end{bmatrix}$

$\sigma_a \bar{A}_1' X_{12} = 0$  and  $\sigma_a \bar{A}_1' X_{12} + -s' + f = 0 + 0 + (-20) = -20$

$\sigma_a \bar{A}_2' X_{22} = 0$  and  $\sigma_a \bar{A}_2' X_{22} + -t' + g = 0 + 0 + (-51.25) = -51.25$

$\sigma_a A_3' = [1 \ 1]$  Therefore the -w row entry for  $x_9 = 1$ ; for  $x_{10} = 1$ .

The most negative entry in the -w row is -51.25 so  $T_2$  is the entering column. The column is:

$\bar{A}_1 X_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  or  $\begin{bmatrix} 0 \\ 0 \\ 0 \\ -51.25 \\ 0 \\ 1 \end{bmatrix}$

$\sigma_a \bar{A}_1' X_{12} + f = -51.25$

$\Phi = 0$

$\mu = 1$

The column enters the basis using the normal procedure for multipliers. First the column is altered by multiplying the row of the inverse by the column above to get the entries used in the entering variable column.

Basis Variable	$E_1$	$E_2$	-z	-w	$\Phi_1$	$\mu_1$	Constant	Entering Variable
$E_1$	-1				10	20	10	$\mu_2$ <span style="border: 1px solid black;">20</span>
$E_2$		-1			10	31.25	21.25	31.25
-z			1		20	65	85	65
-w				1			-31.25	-51.25
$\Phi_1$					1		1	0
$\mu_1$						1	1	1



This results in:

Basis Variable	$E_1$	$E_2$	$-z$	$-w$	$\Phi_1$	$\mu_1$	Constants
$\mu_2$	.05				.5	1	.5
$E_2$	1.563	-1			-5.625	0	5.625
$-z$	3.25		1		-12.5	0	52.500
$-w$	-2.563			1	25.625	51.25	-5.625
$\Phi_1$	0				1	0	1
$\mu_1$	.05				.5	0	.5

$$\sigma_b \begin{bmatrix} -2.563 & 0 & 0 & 1 \end{bmatrix}; \quad -s' = 25.625; \quad -t' = 51.25$$

$$\sigma_b \bar{A}'_1 = \begin{bmatrix} -.563 & -1.063 & 0 & 0 \end{bmatrix}$$

$$\sigma_b \bar{A}'_2 = \begin{bmatrix} .438 & -1.313 & 0 & 0 \end{bmatrix}$$

$$\sigma_b A'_3 = \begin{bmatrix} -1.563 & 1 \end{bmatrix}$$

Solving the following sub-programs:

$x_1$	$x_2$	$x_5$	$x_6$	Constants
1	1	1		10
	2		1	5
-0.563	-.438	0	0	$w_1^2$



and

$x_3$	$x_4$	$x_7$	$x_8$	Constants
1	1	1		20
	1		1	5
.438	-1.313	0	0	$\frac{2}{w_2}$

results in

$$X_{13} = \begin{bmatrix} 7.5 \\ 2.5 \\ 0 \\ 0 \end{bmatrix} \text{ and } X_{23} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix}$$

Then  $\sigma_b \bar{A}'_1 X_{13} = -6.875$  or  $\sigma_b \bar{A}'_1 X_{13} + (-s') + f = -6.875 + 25.625 - 20 = -1.25$ ;  
 $\sigma_b \bar{A}'_2 X_{23} = -6.563$ ;  $\sigma_b \bar{A}'_2 X_{23} + (-t') + g = -6.563 + 51.25 - 51.25 = -6.563$ ;  
 $\sigma_b A'_3 = [-1.563 \ 1]$ ;  $-w$  entry for  $x_9 = -1.563$ ; and  $x_{10} = 1$ .

The variable entering is the one which has the most negative entry of the  $-w$  row from  $S_3$ ,  $T_3$ ,  $x_9$ ,  $x_{10}$ . These entries are respectively  $-1.25$ ,  $-6.563$ ,  $-1.563$ ,  $1$ . Thus the entering variable is  $\mu_3$  which is connected with  $T_3$ . The column is made up as follows:

$$T_3 = \bar{A}_2 X_{23} = \begin{bmatrix} 5 \\ 1.25 \\ -5 \end{bmatrix}$$

$$-w \text{ row entry} = \sigma_b \bar{A}'_2 X_{23} + g = -6.563 - 51.25 = -57.813$$

$$\Phi \text{ row entry} = 0$$

$$\mu \text{ row entry} = 1$$

Therefore the entering column is  $\begin{bmatrix} 5 \\ 1.25 \\ -5 \\ -57.813 \\ 0 \\ 1 \end{bmatrix}$



and when changed to the current inverse the program becomes:

Basis Variable	$E_1$	$E_2$	$-z$	$-w$	$\Phi_1$	$\mu_1$	Constants	Entering Variable
$\mu_2$	-.05				.5	1	.5	$\mu_3$ <u>.75</u>
$E_2$	1.563	-1			-5.625	0	5.625	6.563
$-z$	1.182		1		-12.50	0	52.25	11.25
$-w$	-2.563			1	25.625	51.25	-5.625	-6.563
$\Phi_1$	0				1	0	1	0
$\mu_1$	.05				-.5	0	.5	.25

which solves to the matrix below without the entry in the entering variable column.

Basis Variable	$E_1$	$E_2$	$-z$	$-w$	$\Phi_1$	$\mu_1$	Constants	Entering Variable
$\mu_3$	-.067				.667	1.333	.667	$x_9$ -.067
$E_2$	2	-1			-10	-8.75	1.25	<u>2</u>
$-z$	4		1		-20	-15	45	4
$-w$	-3			1	30	60	-1.25	-2
$\Phi_1$	0				1	0	1	0
$\mu_1$	.067				-.667	-.333	.333	.067



$$\sigma_c = [-3 \ 0 \ 0 \ 1] ; \quad -s' = 30 ; \quad -t' = 60$$

$$\sigma_c \bar{A}'_1 = [-1 \ -1.5 \ 0 \ 0]$$

$$\sigma_c \bar{A}'_2 = [0 \ -1.75 \ 0 \ 0]$$

$$\sigma_c A'_3 = [-2 \ 1]$$

Solving the sub-programs,

$$X_{14} = \begin{bmatrix} 7.5 \\ 2.5 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad X_{24} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix}$$

Solving for the -w row values we get,

$$\sigma_c \bar{A}'_1 X_{14} + (-s') + f = -11.25 + 30 - 20 = -1.25$$

$$\sigma_c \bar{A}'_2 X_{24} + (-t') + g = -8.75 + 60 - 51.25 = 0$$

and the values in  $\sigma_c A'_3$ , where  $x_9 = -2$  and  $x_{10} = 1$ .

The most negative value is -2 which tells us that the column  $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$  enters.

This column has no entries in the  $\Phi$  or  $\mu$  rows, so the entering column is  $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

and when adjusted to suit the inverse becomes the right hand column in the above matrix.



Solution to the above matrix:

Basic Variable	$E_1$	$E_2$	$-z$	$-w$	$\Phi_1$	$\mu_1$	Constants
$\mu_3$	0	-.033			.333	1.042	.708
$x_9$	1	-.50			-5	-1.25	.625
$-z$	0	2	1			2.5	42.5
$-w$	-1	-1		1	20	51.25	0
$\Phi_1$	0	0			1	0	1
$\mu_1$	0	.033			-.033	-.042	-292

$$\sigma_d = [-1 \quad -1 \quad 0 \quad 1]; \quad -s' = 20; \quad -t' = 51.25$$

$$\sigma_d \bar{A}'_1 = [0 \quad 0 \quad 0 \quad 0]$$

$$\sigma_d \bar{A}'_2 = [0 \quad 0 \quad 0 \quad 0]$$

$$\sigma_d \bar{A}'_3 = [0 \quad 0]$$

Therefore

$$\sigma_d \bar{A}'_1 X_{14} = 0 \quad \text{and} \quad \sigma_d \bar{A}'_2 X_{24} = 0$$

Then

$$\sigma_d \bar{A}'_1 X_{14} + (-s') + f = 0 + 20 - 20 = 0$$

and

$$\sigma_d \bar{A}'_2 X_{24} + (-t') + g = 0 + 51.25 - 51.25 = 0$$

Therefore there are no columns associated with variables in the problems which have negative entries in the  $-w$  row. Also  $-w = 0$ , and the basic feasible solution has been found. This is  $\mu_3 = .708$ ;  $x_9 = .625$ ;  $-z = 42.5$ ;  $\Phi_1 = 1$ ; and  $\mu_1 = .292$ . This is the end of the Phase I procedure and the normal Phase II procedure used. The  $-w$  row and  $-w$  column



may now be dropped and the multipliers are now determined from the -z row. Also the regular  $\bar{A}_i$  matrices are used. The remainder of the Phase II solution is given without elaborating on the procedure.

Basic Variable	$E_1$	$E_2$	-z	$\Phi_1$	$\mu_1$	Constants	Entering Variable
							$\Phi_5$
$\mu_3$	0	-.033		.333	1.042	.708	.292
$x_9$	1	-.5		-5	-4.375	.625	-3.125
-z	0	2	1	0	2.5	42.5	-5
$\Phi_1$	0	0		1	0	1	[1]
$\mu_1$	0	.033		-.333	-.042	.292	-.292

$$\pi^0 = [0 \ 2 \ 1]; \quad -s^0 = 0; \quad -t^0 = 2.5$$

$$\pi^0 \bar{A}_1 = [0 \ -2 \ 0 \ 0]$$

$$\pi^0 \bar{A}_2 = [0 \ -.5 \ 0 \ 0]$$

$$\pi^0 \bar{A}_3 = [0 \ 2]$$

The solutions to the sub-programs are:

$$X_{15} = \begin{bmatrix} 0 \\ 2.5 \\ 7.5 \\ 0 \end{bmatrix} \quad \text{and} \quad X_{25} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix}$$

Then

$$\pi^0 \bar{A}_1 X_{15} = -5 \quad \text{and} \quad \pi^0 \bar{A}_1 X_{15} - s^0 = -5 + 0 = -5$$

$$\pi^0 \bar{A}_2 X_{25} = -2.5 \quad \text{and} \quad \pi^0 \bar{A}_2 X_{25} - t^0 = -2.5 + 2.5 = 0$$

Entry in -z row of  $x_9 = 0$ ; of  $x_{10} = 2$ , so then entering variable is  $\Phi_5$ ,



and the entering column before alteration to the inverse is  $\begin{bmatrix} 2.5 \\ 1.25 \\ -7.50 \\ 1 \\ 0 \end{bmatrix}$

The solution:

Basis Variable	$E_1$	$E_2$	$-z$	$\Phi_1$	$\mu_1$	Constants
$\mu_3$	0	-.033	0	.042	1.042	.417
$x_9$	1	-.5	0	-1.875	-4.375	3.75
$-z$	0	2	1	5	2.5	47.5
$\Phi_5$	0	0	0	1	0	1
$\mu_1$	0	.033	0	-.042	-.042	.583

$$\pi^1 = 0 \ 2 \ 1 ; \ -s^1 = 5; \ -t^1 = 2.5$$

The above are precisely the same multipliers obtained in the last iteration of the procedure using only the Phase II methods. This occurred at the optimum solution, so we have the optimum solution. It is  $\mu_3 = .417$ ;  $x_3 = 3.75$ ;  $-z = 47.5$ ;  $\Phi_5 = 1$ ; and  $\mu_1 = .583$ . In this case, the solution to the main program is using .583 of  $X_{21}$ , .417 of  $X_{23}$ , and all of  $X_{15}$ . As well,  $x_9 = 3.75$  and  $z = -47.5$ .

$$X_2 = \begin{bmatrix} x_3 \\ x_4 \\ x_7 \\ x_8 \end{bmatrix}; \quad X_{21} = \begin{bmatrix} 15 \\ 5 \\ 0 \\ 0 \end{bmatrix}; \quad X_{23} = \begin{bmatrix} 0 \\ 5 \\ 15 \\ 0 \end{bmatrix}; \quad X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_6 \end{bmatrix}; \quad X_{15} = \begin{bmatrix} 0 \\ 2.5 \\ 7.5 \\ 0 \end{bmatrix}$$

Thus  $x_2 = 2.5$ ;  $x_3 = 8.75$ ;  $x_4 = 5$ ;  $x_5 = 7.5$ ;  $x_7 = 6.25$ ;  $x_9 = 3.75$ , and  $x_1 = x_6 = x_8 = x_{10} = 0$ , which is exactly the same answer as obtained previously.



## 7.2 REPORT ON PROJECT SEMINAR

A seminar was held on December 21, 1965 in the Legislative Building, Government of Alberta, with the purposes of presenting to and discussing with Extension personnel the results of this project. Staff members of the Farm Economics Branch, District Agriculturists from various districts of the province, several of the farmers cooperating in the project, and others who had some interest in the technique were in attendance. Dr. G. R. Purnell, Supervisor of the Farm Economics Branch, and Dr. F. V. MacHardy, Professor and Head, Department of Agricultural Engineering, University of Alberta, gave summaries of the events leading to the development of the project. The author then summarized the project and the results. The discussion which followed centered on the experience gained from this project and an explanation of the requirements of linear programming to Extension personnel. As well, mention was made of post optimal information which was available but not studied in this project.

The farmers gave their opinion of the project and were generally satisfied with the results. The farmers and the District Agriculturists showed considerable interest in the post optimal information which could have been made available.

Dr. Purnell stated at the conclusion of the seminar that the work begun in this project was going to be continued if at all possible until it was decided whether or not it was feasible to offer an expanded service.





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